



A MATHEMATICAL FORMULATION OF THE JANGID NESTED-DOMAIN COSMOLOGICAL HYPOTHESIS

A PARAMETERIZED MATHEMATICAL-MODELLING FRAMEWORK FOR HIERARCHICAL COSMOLOGICAL DOMAINS

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ABSTRACT:

This paper develops the Jangid Nested-Domain Cosmological Hypothesis (JNDCH) as a mathematical and phenomenological framework rather than as an established physical theory. The central mathematical idea is to represent a collection of effective cosmological domains by an indexed family $\{U_i\}_{i \in I}$, with each domain assigned a differentiable scale factor, an effective spatial-curvature parameter, matter and dark-energy densities, and an expansion rate. The standard Friedmann-type equations are used as the dynamical background for each domain, while a dimensionless nested-domain descriptor is introduced to quantify deviations from a chosen reference cosmology. The manuscript gives explicit definitions of the mathematical objects, states the regularity and normalization assumptions, and establishes elementary propositions concerning the parameterization, identifiability and limiting cases. A probabilistic formulation is provided for questions concerning Earth-like domains, with the independence assumptions stated explicitly. For possible observational testing, a constrained angular-modulation ansatz is introduced and its parameter degeneracies are analyzed. A likelihood-based testing strategy is then specified so that the hypothesis can, in principle, be compared with a reference cosmological model. The work does not claim evidence for additional universes, a parent domain, or a Creator. Its contribution is the construction of a mathematically explicit, testable parameterization that can be developed or rejected through future theoretical and observational work.

KEYWORDS:

MATHEMATICAL COSMOLOGY, MATHEMATICAL MODELLING, FLRW GEOMETRY, DIFFERENTIAL EQUATIONS, DIMENSIONLESS PARAMETERS, PHENOMENOLOGICAL MODEL.

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1. INTRODUCTION

Mathematical cosmology studies cosmological questions through differential geometry, differential equations, dynamical systems, probability and statistical inference. The present work is positioned in that mathematical tradition. The purpose is not to replace the standard cosmological model by assertion, but to construct a precise mathematical language for a specific hypothesis: the observable cosmological region might be one effective member of a larger hierarchy of domains.

Observational data strongly constrain the standard spatially flat Λ CDM model. Planck 2018 parameter analyses found the base Λ CDM model to provide a good description of the CMB power spectra and lensing data, with spatial curvature constrained close to zero in the relevant combined analyses [1]. This paper therefore does not assume that a measurable curvature anomaly has already been observed. Instead, it asks whether a larger-domain hypothesis can be formulated without

contradicting the mathematical structure of an approximately homogeneous and isotropic observable region.

The key methodological requirement is mathematical well-definedness. Every symbol introduced below is assigned a domain and interpretation; every phenomenological parameter is normalized; and every proposed observable effect is accompanied by conditions under which it is identifiable. This distinction is essential because a flexible parameterization can otherwise fit arbitrary residuals without constituting a meaningful scientific model.

2. MATHEMATICAL PRELIMINARIES AND DEFINITIONS

Definition (2.1 Index set). Let I be a nonempty index set. Each $i \in I$ labels one effective cosmological domain. I may be finite or countably infinite for the present formulation.

Definition (2.2 Effective domain). An effective domain U_i is a mathematical object representing a spacetime region or a causally characterized cosmological sector to which a separate effective description is assigned. The notation does not, by itself, assert physical disconnectedness.

Definition (2.3 Hierarchical family). A hierarchical family is a collection $\{U_i\}_{i \in I}$ together with a level map $\ell: I \rightarrow \mathbb{N}$. Domains assigned to different levels may represent progressively broader effective descriptions. The hierarchy is therefore a modelling structure; a literal set inclusion $U_i \subset U_j$ is not assumed unless a specific physical construction supplies it.

Definition (2.4 Spacetime manifold). For a domain i , let M_i be a smooth four-dimensional manifold and let g_i be a Lorentzian metric on M_i . Smoothness is assumed at least to the order required for the curvature tensors and field equations used below.

Definition (2.5 Scale factor). The function $a_i(t) > 0$ is a dimensionless, differentiable scale factor normalized by $a_i(t_0) = 1$ at a chosen reference time t_0 . Its logarithmic derivative defines the domain Hubble parameter.

Definition (2.6 Hubble parameter). $H_i(t) = \dot{a}_i(t)/a_i(t)$, with physical dimension time^{-1} . The present value is $H_{i,0} = H_i(t_0)$.

Definition (2.7 Curvature parameter). k_i is the normalized constant-curvature parameter in the FLRW metric convention used here. With dimensionless radial coordinate r , k_i is dimensionless and takes the sign classes $-1, 0, +1$ after a conventional normalization. More general dimensional conventions are possible but must not be mixed within one derivation.

Definition (2.8 Energy density and pressure). $\rho_i(t)$ denotes total mass-energy density expressed as mass per unit volume, while $p_i(t)$ denotes isotropic pressure. They satisfy the regularity conditions required by the chosen dynamical model.

Definition (2.9 Cosmological term). $\Lambda_i(t)$ denotes an effective cosmological term. The simplest version of the model takes Λ_i constant in time; a time-dependent $\Lambda_i(t)$ is treated only as a future extension.

Definition (2.10 Density parameters). For $H_i \neq 0$, dimensionless density parameters are defined by $\Omega_{m,i} = 8\pi G \rho_{m,i} / (3H_i^2)$, $\Omega_{DE,i} = 8\pi G \rho_{DE,i} / (3H_i^2)$, and $\Omega_{k,i} = -k_i c^2 / (a_i^2 H_i^2)$. For a cosmological constant, $\Omega_{\Lambda,i} = \Lambda_i c^2 / (3H_i^2)$.

Definition (2.11 Reference cosmology). A reference cosmology is a specified model with parameter vector θ_{ref} and corresponding functions $H_{\text{ref}}(z)$, $\Omega_{m,\text{ref}}(z)$, $\Omega_{k,\text{ref}}(z)$, and $\Omega_{DE,\text{ref}}(z)$. The reference must be fixed before parameter estimation; otherwise the proposed deviation parameter can become data-dependent by construction.

Definition (2.12 Phenomenological parameter). A phenomenological parameter is a mathematically defined

quantity introduced to summarize deviations from a reference model without claiming that the parameter is a fundamental constant of nature.

Definition (2.13 Identifiability). A parameter is identifiable within a specified model and dataset if distinct parameter values produce distinct probability distributions for the observables, subject to the stated nuisance parameters and symmetries.

3. GEOMETRICAL MODEL

Assume that, at the scales relevant to the background model, each domain is sufficiently homogeneous and isotropic. The metric is then represented in FLRW form:

$$ds_i^2 = -c^2 dt^2 + a_i^2(t) [dr^2 / (1 - k_i r^2) + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)] \quad (1)$$

Here $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$ is the metric on the unit two-sphere. Equation (1) is an ansatz for the effective geometry of each domain. It is not a derivation of the existence of multiple domains.

Mathematical note. The same mathematical form may describe many physically different models. Consequently, the metric alone cannot distinguish JNDCH from other cosmological scenarios; the proposed domain-index structure and the associated parameterization are what define the present hypothesis.

4. DYNAMICAL EQUATIONS

For each domain, the background expansion is described by the Friedmann equation:

$$H_i^2 = (8\pi G/3)\rho_i - k_i c^2 / a_i^2 + \Lambda_i c^2 / 3 \quad (2)$$

The corresponding acceleration equation is

$$\ddot{a}_i / a_i = -(4\pi G/3)(\rho_i + 3p_i/c^2) + \Lambda_i c^2 / 3 \quad (3)$$

and the continuity equation is

$$\dot{\rho}_i + 3H_i(\rho_i + p_i/c^2) = 0 \quad (4)$$

when the effective stress-energy sector is separately conserved. Equations (2)–(4) are standard relativistic cosmological relations used here as the mathematical background. JNDCH modifies neither general relativity nor these equations at this stage; it assigns them domain-dependent effective parameters.

Proposition (Consistency relation). If the total density is decomposed into matter, radiation and dark-energy components and the definitions of the corresponding Ω parameters are used consistently, then the Friedmann equation implies a closure relation of the form $\Omega_m + \Omega_r + \Omega_{DE} + \Omega_k = 1$. For a cosmological-constant component, Ω_{DE} may be replaced by Ω_{Λ} .

The redshift variable is defined by

$$1+z = a_i(t_0)/a_i(t) = 1/a_i(t) \quad (5)$$

under the normalization $a_i(t_0) = 1$. Thus $z \geq 0$ corresponds to earlier epochs in the usual expanding-background interpretation.

5. FORMAL DEFINITION OF THE JNDCH

Definition (JNDCH model). A JNDCH model is a tuple $M = (I, \{M_{i,g_i,a_i,p_i,\Lambda_i}\}_{i \in I}, \theta)$, where I is an index set

of effective domains and Θ is a finite-dimensional parameter vector controlling the specified relation between domain-level quantities and the reference cosmology.

The core hypothesis can then be stated mathematically as follows:

Proposition (Nested-domain hypothesis). There exists at least one admissible indexed family of effective cosmological domains for which the observable domain is represented by an element U_0 of the family, while the full family contains additional indices $i \neq 0$ that are not necessarily directly observable from U_0 . The hypothesis becomes scientifically testable only when a specified mapping from domain-level structure to observables is supplied.

This definition is intentionally conditional. It does not assert that such additional domains have been observed.

6. NESTED-DOMAIN PARAMETERIZATION

Let the domain descriptor be

$$N_i = \alpha \Delta\Omega_{\{m,i\}} + \beta \Delta\Omega_{\{k,i\}} + \gamma \Delta\Omega_{\{DE,i\}} \quad (6)$$

$$\begin{aligned} \Delta\Omega_{\{m,i\}} &= \Omega_{\{m,i\}} - \Omega_{\{m,ref\}}, & \Delta\Omega_{\{k,i\}} &= \Omega_{\{k,i\}} - \Omega_{\{k,ref\}}, \\ \Delta\Omega_{\{DE,i\}} &= \Omega_{\{DE,i\}} - \Omega_{\{DE,ref\}} \end{aligned} \quad (7)$$

where α , β and γ are dimensionless weights. For identifiability, a convention must be imposed, for example $\alpha^2 + \beta^2 + \gamma^2 = 1$, together with a sign convention such as $\alpha \geq 0$. Other normalization choices are possible, but they must be declared before fitting.

Proposition (Reference limit). If $\Omega_{\{m,i\}} = \Omega_{\{m,ref\}}$, $\Omega_{\{k,i\}} = \Omega_{\{k,ref\}}$, and $\Omega_{\{DE,i\}} = \Omega_{\{DE,ref\}}$, then $N_i = 0$.

Proposition (Non-equivalence of $N_i = 0$). The converse is not generally true. $N_i = 0$ does not imply that all three deviations vanish because positive and negative weighted deviations can cancel. Therefore N_i is a scalar summary, not a complete coordinate system for domain differences.

Mathematical note. This proposition prevents overinterpretation of N_i . If the scientific question requires recovery of each deviation separately, the vector $\Delta\Omega_i = (\Delta\Omega_{\{m,i\}}, \Delta\Omega_{\{k,i\}}, \Delta\Omega_{\{DE,i\}})$ must be retained instead of reducing it to a single scalar.

7. ALTERNATIVE VECTOR FORMULATION

For mathematical clarity, define

$$\Delta\Omega_i = (\Delta\Omega_{\{m,i\}}, \Delta\Omega_{\{k,i\}}, \Delta\Omega_{\{DE,i\}})^T, \quad w = (\alpha, \beta, \gamma)^T \quad (8)$$

$$N_i = w^T \Delta\Omega_i \quad (9)$$

This form shows that N_i is the projection of a three-dimensional deviation vector onto a chosen weight vector. The null space of w contains all deviations that are invisible to N_i . Consequently, the scalar parameter is useful for a compact phenomenological model but cannot encode every possible domain difference.

Proposition (Null-space condition). If $w \neq 0$, the set $\{\Delta\Omega_i : w^T \Delta\Omega_i = 0\}$ is a two-dimensional linear subspace of the three-dimensional parameter space. Therefore infinitely many distinct domain parameter vectors can share $N_i = 0$.

8. TESTABLE OBSERVABLE MODEL

To connect the abstract parameterization to data, let $X(z, \hat{n})$ be an observable depending on redshift z and sky direction $\hat{n}$. A first-order modulation model is

$$X(z, \hat{n}) = X_{ref}(z) [1 + \varepsilon N_0 f(\hat{n})] \quad (10)$$

where ε is a dimensionless coupling amplitude, N_0 is the descriptor of the observable domain, and f is a predetermined angular function. To avoid a normalization degeneracy with X_{ref} , impose

$$\int_{\{S^2\}} f(\hat{n}) d\Omega = 0, \quad \int_{\{S^2\}} f^2(\hat{n}) d\Omega = 1 \quad (11)$$

A concrete basis can be obtained from spherical harmonics. For example, f may be chosen as a normalized real spherical harmonic $Y_{\{\ell m\}}$ with $\ell \geq 1$. The choice of f must be fixed independently of the data or justified by a physical mechanism.

Proposition (First-order null model). If $\varepsilon = 0$ or $N_0 = 0$, equation (10) reduces exactly to $X(z, \hat{n}) = X_{ref}(z)$. Thus the reference cosmology is a nested null model of the proposed modulation.

Proposition (Identifiability condition). If f is identically zero, or if its functional form is not fixed independently of the data, εN_0 is not identifiable from equation (10). Even with nonzero f , ε and N_0 appear only through their product unless one of them is fixed or independently constrained. A data analysis must therefore parameterize the product $A = \varepsilon N_0$ or supply an external normalization.

9. STATISTICAL FORMULATION FOR EMPIRICAL TESTING

Suppose measurements $y \in \mathbb{R}^n$ have covariance matrix C , with C symmetric positive definite. Let $\mu(\theta)$ denote the model prediction. The Gaussian log-likelihood is equivalent to minimizing

$$\chi^2(\theta) = [y - \mu(\theta)]^T C^{-1} [y - \mu(\theta)] \quad (12)$$

The reference model and JNDCH extension should be compared using the same dataset, nuisance-parameter treatment and likelihood definition. A parameter-count penalty may be incorporated through information criteria such as

$$AIC = \chi^2_{min} + 2k, \quad BIC = \chi^2_{min} + k \ln n \quad (13)$$

where k is the number of fitted parameters and n is the number of observations. These criteria are not themselves evidence for the hypothesis; they are model-comparison tools. Bayesian evidence or cross-validation may be used as alternatives.

Mathematical note. A scientifically meaningful test requires pre-specified observables, parameter priors or fitting ranges, treatment of systematic errors, and a decision rule. Fitting an arbitrary angular function after inspecting residuals would substantially weaken the falsifiability of the model.

10. PROBABILITY OF EARTH-LIKE DOMAINS

Definition (Earth-like criterion). Let E be a precisely specified event, such as a domain containing at least one

planet satisfying a chosen vector of habitability criteria. The criteria must be fixed before the probability is evaluated.

If N candidate domains are assumed independent and each has the same probability $p=P(E)$, then

$$P(\geq 1 \text{ occurrence}) = 1 - (1-p)^N \quad (14)$$

Proposition (Monotonicity). For $0 \leq p \leq 1$ and integer $N \geq 1$, the probability in equation (14) is nondecreasing in both p and N .

If the probabilities differ by domain, $p_i = P(E_i)$, independence gives

$$P(\geq 1 \text{ occurrence}) = 1 - \prod_{i=1}^N (1-p_i) \quad (15)$$

and if the domains are correlated, the product formula generally fails. Therefore equation (14) is a conditional probability model, not evidence that another Earth exists.

11. LIMITING CASES AND INTERNAL CHECKS

Proposition (Single-domain limit). For $N=1$ in the probabilistic formulation, equation (14) gives $P(\geq 1 \text{ occurrence}) = p$, as expected.

Proposition (Zero-deviation limit). For $\Delta\Omega_i=0$, the proposed nested-domain descriptor satisfies $N_i=0$ and the observable modulation in equation (10) reduces to the reference model.

Proposition (Small-coupling limit). For $|\epsilon N_0 f| \ll 1$, equation (10) is a first-order perturbation about X_{ref} . Higher-order terms may be required if the fitted amplitude is not small.

These checks are necessary because they connect the proposed model to known limiting behaviour and reduce the risk of introducing parameters whose mathematical interpretation is ambiguous.

12. RELATION TO EXISTING COSMOLOGICAL MODELS

The JNDCH framework is not identical to de Sitter or Einstein-de Sitter cosmology. Those are specific solutions or model classes characterized by particular matter, curvature and cosmological-constant assumptions. JNDCH instead introduces an indexed family of effective domains and asks whether domain-to-domain differences can be parameterized and constrained.

It is also distinct from eternal-inflation and higher-dimensional/brane scenarios. Those frameworks can provide physical mechanisms for multiple regions or effective sectors, whereas JNDCH presently supplies a phenomenological mathematical structure without selecting a unique microphysical mechanism. This distinction is important: JNDCH should not be interpreted as a derivation from inflation, string theory, or brane cosmology.

13. OBSERVATIONAL PROGRAMME

13.1 EXPANSION HISTORY

Estimate $H(z)$, distance-redshift relations, or related background observables and test whether the specified

JNDCH extension improves the likelihood relative to the reference model after accounting for additional parameters.

13.2 LARGE-SCALE ANGULAR STRUCTURE

Test a pre-specified $f(\hat{n})$ against CMB, supernova, BAO or other directional datasets. Foregrounds, survey masks, calibration and look-elsewhere effects must be included in the analysis.

13.3 PRIMORDIAL SIGNATURES

If a dynamical completion of JNDCH predicts correlations in primordial perturbations, the prediction should be translated into a power spectrum, correlation function or other statistic before comparison with data.

13.4 NUMERICAL DEVELOPMENT

A future dynamical theory should derive domain-dependent parameters from an action, coupled differential equations, boundary conditions or a stochastic process rather than assigning them independently by definition.

14. LIMITATIONS

- No observation currently establishes the existence of a parent cosmic domain or additional universe.
- The scalar N_i is a phenomenological projection and is not a fundamental constant.
- The current model does not provide a microphysical mechanism that generates multiple domains.
- The angular modulation is an explicitly proposed ansatz and is not an established cosmological law.
- Parameter identifiability can fail if ϵ and N_0 are both free or if f is chosen adaptively from the data.
- The Earth-like probability model depends on astrophysical definitions, domain counts and independence assumptions that are not currently established.
- The philosophical discussion of a Creator or Supreme Ordering Principle is outside the empirical content of the mathematical model.

15. PHILOSOPHICAL INTERPRETATION AND SCIENTIFIC BOUNDARY

Mathematics provides a language for expressing regularities; it does not by itself determine a metaphysical explanation for why those regularities exist. The possibility of interpreting cosmic mathematical order as compatible with a Creator is therefore a philosophical or theological interpretation, not a consequence proved by equations (1)–(15). The present paper deliberately keeps that interpretation separate from the empirical model.

16. FUTURE MATHEMATICAL RESEARCH

1. Derive a covariant action or effective field theory whose solutions naturally generate the indexed domain structure.
2. Replace the scalar descriptor N_i by a

higher-dimensional parameter space when required by observations.

3. Develop a dynamical system for the coupled evolution of $a_i(t)$, density components and any domain-coupling variables.
4. Study stability, fixed points and perturbations of the resulting system.
5. Develop Bayesian or likelihood-based inference with public cosmological datasets.
6. Investigate whether a mathematically consistent stochastic process can generate the domain ensemble without introducing arbitrary distributions.

17. CONCLUSION

The Jangid Nested-Domain Cosmological Hypothesis has been reformulated here as a mathematically explicit phenomenological framework. Its central objects—indexed domains, smooth spacetime manifolds, scale factors, Hubble parameters, curvature parameters, density parameters, a dimensionless deviation descriptor and a constrained observable ansatz—have been defined before being used. Elementary propositions establish the reference limit, the non-uniqueness of the scalar descriptor and the identifiability conditions of the proposed modulation.

The principal scientific claim is intentionally limited: JNDCH provides a mathematical parameterization with which a nested-domain hypothesis can be investigated. It does not establish additional universes, a parent domain, or a metaphysical Creator. The next meaningful step is therefore not stronger wording but stronger mathematics: a dynamical derivation, parameter identifiability analysis and comparison with observational data. If those developments fail to produce a distinguishable prediction, the model should be regarded as a descriptive framework rather than a physical theory.

AUTHOR DECLARATION

The author declares that this manuscript presents a conceptual mathematical and phenomenological hypothesis. No claim is made that JNDCH is an experimentally established cosmological theory. The equations adopted from standard cosmology are used as mathematical background relations, while the nested-domain parameterization and observational ansatz are proposed constructs requiring future theoretical and empirical validation.

PUBLICATION-STATUS NOTE:

This is a mathematically strengthened research-manuscript draft. Peer-review acceptance cannot be guaranteed by formatting or rewriting alone; the model must ultimately withstand expert review and, ideally, quantitative validation.

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