A NOTE ON LOG LOG LAW FOR SUBSEQUENCES OF WEIGHTED SUMS

GOOTY DIVANJI [1]

[1] DEPARTMENT OF STATISTICS, MANASAGANGOTHRI UNIVERSITY OF MYSORE, MYSORE - 570 006 - KARNATAKA – INDIA.

ABSTRACT:

Let $\{X_n, n \geq 1\}$ be a sequence of independent and identically distributed random variables with a common distribution function F. When F belongs to the domain of partial attraction of a semistable law, with index α, $0 < \alpha < 1$, We study Chover's form of law of iterated logarithm for subsequences of weighted sums and extended to boundary crossing problem.

KEYWORDS:

LAW OF ITERATED LOGARITHM, SEMISTABLE LAW, DOMAIN OF PARTIAL ATTRACTION, WEIGHTED SUMS, BOUNDARY CROSSING PROBLEM.

MSC: primary 60F10 and 60F15

Subject: Statistics – Probability Theory

1. INTRODUCTION

Let $\{X_n, n \geq 1\}$ be a sequence of independent and identically distributed (i.i.d) random variables (r.v.s) with a common distribution function F. Let BV [0, 1] be the sets of all continuous bounded variation functions over [0,1]. Set $S_n = \sum_{k=1}^{n} X_k$, $n \geq 1$ and $T_n = \sum_{k=1}^{n} f\left(\frac{k}{n}\right) X_k$, where f is a member of BV [0,1]. Let $\{n_k, k \geq 1\}$ be a strictly increasing subsequence of positive integers such that $\frac{n_{k+1}}{n_k} \to r(\geq 1)$, as $k \to \infty$. Kruglov (1972) established that, if there exist sequences (a_k) and (b_k) of real constants, $b_k \to \infty$, as $k \to \infty$, such that

$$\lim_{k\to\infty} P\left(\frac{S_{n_k}}{b_k} - a_k \leq x\right) = G_\alpha(x) \tag{1}$$

at all continuity points x of G_α, then G_α is necessarily a semistable d.f with characteristic exponent α, $0 < \alpha \leq 2$. When $\alpha = 2$, semistable d.f. is normal d.f.

We assume that $a_k = 0$ in (1). When $\alpha<1$, a_k can always be chosen to be zero. When $\alpha > 1$, a_k becomes $n_k EX_1$. Here one can make $a_k = 0$ by shifting EX_1 to zero. Consequently, the condition $a_k = 0$ is no condition at all when $\alpha \neq 1$, $0 < \alpha < 2$. However, when $\alpha = 1$, this assumption restricts only to symmetric d.f.s, $F \in DP(1)$.

When $E X_n^2 < \infty$, Allan Gut (1986) established the classical law of iterated logarithm (LIL) for geometrically fast increasing subsequence of (S_n). In fact, he established that

$$\limsup_{k\to\infty} \frac{S_{n_k}}{\sqrt{2 n_k \log\log n_k}} = \begin{cases} 1 \text{ a.s,} & \text{if } \limsup_{k\to\infty} \frac{n_{k+1}}{n_k} < \infty \\ \varepsilon^* \text{ a.s,} & \text{if } \liminf_{k\to\infty} \frac{n_{k+1}}{n_k} > 1 \end{cases}$$

where $\varepsilon^* = \inf\left\{\varepsilon > 0 : \sum_{k=1}^{\infty} (\log n_k)^{-\frac{\varepsilon^2}{2}} < \infty\right\}$. Observe that, when $n_k = 2^{2^{\cdot^{\cdot^{k}}}}$, then $\varepsilon^* = 0$ and we have $\operatorname*{Lim\,sup}_{k\to\infty} \frac{S_{n_k}}{\sqrt{2 n_k \log\log n_k}} = 0$ a. s. I.e., for such cases the norming sequence $\sqrt{2 n_k \log\log n_k}$ will not be precise enough to give almost sure bound for (S_{n_k}). In general, whenever $\frac{n_{k+1}}{n_k} \to \infty$, as $k \to \infty$, Rainer Schwabe and Allan Gut (1996) have pointed out that $\sqrt{2 n_k \log\log n_k}$ is no longer the proper normalizing sequence and it has to be replaced by $\sqrt{2 n_k \log k}$.

When $n_k = n$, Chover (1966) observed that in the case of stable r.v.s, LIL involving Lim sup cannot be obtained under linear normalization and that it is possible in power normalization only. In fact, when X_n's are i.i.d. symmetric stable r.v.s, he established the LIL for (S_n), by normalizing in the power. I.e., $\operatorname*{Lim\,sup}_{n\to\infty} \left|\frac{S_n}{n^{\frac{1}{\alpha}}}\right|^{\frac{1}{\log\log n}} = e^{\frac{1}{\alpha}}$ a.s. and for r.v.s which are in the domain of attraction of a stable law, Peng and Cheng Qi (2003) obtained Chover's type LIL for weighted sums, where the weights are belongs to BV [0, 1]. Many authors studied the non - trivial limit behaviour for different weighted sums. See Peng and Qi (2003) and references therein.

When the underlying r.v.s are in the domain of partial attraction of a semistable law, under Kruglov's set-up, denoted as $F \in DP(\alpha)$, $0 < \alpha < 1$, Gooty Divanji and Kokkada Vidyalaxmi (2011) obtained Chover's form of LIL for (T_n).

Allan Gut (1986) and Rainer Schwabe and Allan Gut (1996) observations motivated us to study the Chover's form of LIL for (T_{n_k}), when $F \in DP(\alpha)$, $0 < \alpha < 1$ and extended to boundary crossing problem.

In the next section we present some lemmas and main results. In the last section we discuss boundary crossing problem. In the process i.o, a.s and s.v. mean 'infinitely often', 'almost surely' and 'slowly varying' respectively. C, ε, k and n with or without a super script or subscript denote positive constants with k and n confined to be integers. In the sequel, observe that when $\alpha < 1$, a_k can always be chosen to be zero.

2. LEMMAS AND MAIN RESULTS

LEMMA 1

Let $F \in DP(\alpha)$, $0 < \alpha < 1$. Then there exists s. v. function L, such that

$$\lim_{x\to\infty} \frac{x^{\alpha}(1 - F(x))}{L(x)} = 1.$$

LEMMA 2

Let $F \in DP(\alpha)$, $0 < \alpha < 1$ and let $B_n = \inf\left\{x > 0 : 1 - F(x) \geq \frac{1}{n}\right\}$. Then $B_n = n^{1/\alpha} l(n)$,

where l is a function s. v. at ∞.

The above lemmas can be referred to Divanji and Vasudeva (1989).

LEMMA 3

Let L be any s. v. function and let (x_n) and (y_n) be sequence of real constants tending to ∞ as $n \to \infty$. Then for any $\delta > 0$, $\lim_{n\to\infty} y_n^{\delta} \frac{L(x_n y_n)}{L(x_n)} = \infty$ and $\lim_{n\to\infty} y_n^{-\delta} \frac{L(x_n y_n)}{L(x_n)} = 0$.

This lemma can be referred to Drasin and Seneta (1986).

LEMMA 4

Let $F \in DP(\alpha)$, $0 < \alpha < 1$. Then $\lim_{n\to\infty} \frac{P(T_n \geq x_n B_n)}{n P(X \geq x_n B_n)} = 1$.

This lemma can be referred to Gooty Divanji and Kokkada Vidyalaxmi (2011).

THEOREM 1

Let $\{X_n, n \geq 1\}$ be a sequence of i.i.d. r.v.s with a common distribution function F and assume that $F \in DP(\alpha)$, $0 < \alpha < 1$. Let $\{B_n\}$ be a sequence of constants as defined in Lemma 2, $T_{n_k} = \sum_{i=1}^{n_k} f\left(\frac{i}{n_k}\right) X_{n_k}$, where f is a member of BV [0,1]. Let (n_k) be an integer subsequence such that

$$\operatorname*{Lim\,inf}_{k\to\infty} \frac{n_{k+1}}{n_k} > 1. \qquad \text{Then} \qquad \operatorname*{Lim\,sup}_{k\to\infty} \left(\frac{T_{n_k}}{B_{n_k}}\right)^{\frac{1}{\log\log n_k}} = e^{\frac{\varepsilon^*}{\alpha}} \text{ a.s}, \tag{2}$$

where $\varepsilon^* = \inf\left\{\varepsilon > 0 : \sum_{k=k_o}^{\infty} (\log n_k)^{-\varepsilon} < \infty\right\}$.

In particular, if $\operatorname*{Lim}_{k\to\infty} \frac{n_{k+1}}{n_k} = \infty$, as $k \to \infty$ and $\varepsilon^* = 0$ then $\operatorname*{Lim\,sup}_{k\to\infty} \left(\frac{T_{n_k}}{B_{n_k}}\right)^{\frac{1}{\log k}} = e^{\frac{1}{\alpha}}$ a.s.

PROOF

To prove the assertion, it suffices to show for any $\varepsilon \in (0, 1)$, that

$$P\left(T_{n_k} \geq B_{n_k} (\log n_k)^{\frac{\varepsilon^*+\varepsilon}{\alpha}} \text{ i.o}\right) = 0 \tag{3}$$

and

$$P\left(T_{n_k} \geq B_{n_k} (\log n_k)^{\frac{\varepsilon^*-\varepsilon}{\alpha}} \text{ i.o}\right) = 1 \tag{4}$$

To prove (3), let $M_k = \left\{T_{n_k} \geq B_{n_k} (\log n_k)^{\frac{\varepsilon^*+\varepsilon}{\alpha}}\right\}$ and $y_n = B_n (\log n)^{\frac{\varepsilon^*+\varepsilon}{\alpha}}$. By the above Lemma 4, one can find a C_1 such that, $P(M_k) \leq C_1 n_k P\left(X \geq y_{n_k}\right)$ and using Lemma 1, one can find a k_1 (>0) such that for all $k \geq k_1$,

$$P(M_k) \leq C_1 n_k y_{n_k}^{-\alpha} L\left(y_{n_k}\right) = C_1 n_k \frac{L\left(B_{n_k}\right)}{B_{n_k}^{\alpha} (\log n_k)^{(\varepsilon^*+\varepsilon)}} \frac{L\left(x_{n_k}\right)}{L\left(B_{n_k}\right)}.$$

Applying Lemma 3 with $\delta = \frac{\varepsilon}{2}$, one can find a k_2 ($\geq k_1$) such that for all k ($\geq k_2$),

$P\left(M_k\right) \leq C_2 \left(\log\ n_k\right)^{-(\varepsilon^* + \frac{\varepsilon}{2})}$, for some $C_2 > 0$. Consequently $\sum_{k=k_2}^{\infty} P(M_k) < \infty$ and (3) follows from the definition of ε^* and by the Borel - Cantelli Lemma.

Define, for large k $m_k = \min\left\{ j : n_j \geq \beta^{(k-1)^{\delta}} \right\}$, (5)

where $\beta > 1$ and $\delta > 0$ and from the relation $T_{n_k} = T_{n_k} - T_{n_{k-1}} + T_{n_{k-1}}$, $k \geq 1$, and in order to establish (4), it is enough if we show that $\varepsilon \in (0, 1)$, that

$$P\left(T_{n_{m_k}} - T_{n_{m_{k-1}}} \geq 2B_{n_{m_k}} \left(\log n_{m_k}\right)^{\frac{\varepsilon^*-\varepsilon}{\alpha}} \text{ i.o}\right) = 1 \tag{6}$$

and

$$P\left(T_{n_{m_{k-1}}} \geq B_{n_{m_k}} \left(\log n_{m_k}\right)^{\frac{\varepsilon^*-\varepsilon}{\alpha}} \text{ i.o}\right) = 0 \tag{7}$$

Define $z_n = B_n (\log n)^{\frac{(\varepsilon^*-\varepsilon)}{\alpha}}$ and $Q_k = \left(T_{n_{m_k}} - T_{n_{m_{k-1}}} \geq z_{n_{m_k}}\right)$, $k \geq 1$. Note that

$T_{n_{m_k}} - T_{n_{m_{k-1}}} \overset{d}{=} T_{n_{m_k} - n_{m_{k-1}}}$, k ≥ 1. By Lemma 4, one can find a constant $C_3 > 0$ and k_3 such that for all k (≥ k_3),

$$P(Q_k) \geq C_3 (n_{m_k} - n_{m_{k-1}}) P(X \geq 2 z_{n_{m_k}}) = C_3\, n_{m_k} \left(1 - \frac{n_{m_{k-1}}}{n_{m_k}}\right) P(X \geq 2z_{n_{m_k}}).$$

Since $\operatorname*{Lim\,inf}_{k\to\infty} \frac{n_{k+1}}{n_k} > 1$ implies that there exists λ<1 such that $\frac{n_{m_{k-1}}}{n_{m_k}} < \lambda < 1$ for all k≥k_3. $P(Q_k) \geq C_4 n_{m_k} P(X \geq 2z_{n_{m_k}})$, for some C_4 >0.

Now following the steps similar to those used to get an upper bound of $P(M_k)$, one can find a k_4 such that for all k (≥ k_4), $P(Q_k) \geq C_5 (\log n_k)^{-(\varepsilon^* - \frac{\varepsilon}{2})}$, for some C_5 > 0. Hence $\sum_{k=k_5}^{\infty} P(Q_k) = \infty$. In view of the fact that Q_k's are mutually independent and by applying the Borel - Cantelli Lemma, (6) is established.

Observe that $$P\left(T_{n_{m_{k-1}}} \geq B_{n_{m_k}} (\log n_{m_k})^{\frac{\varepsilon^*-\varepsilon}{\alpha}}\right) = P\left(T_{n_{m_{k-1}}} \geq B_{n_{m_{k-1}}} \frac{B_{n_{m_k}}}{B_{n_{m_{k-1}}}} (\log n_{m_k})^{\frac{\varepsilon^*-\varepsilon}{\alpha}}\right)$$

Again, by Lemma 4, one can find a constant C_6 and k_5 such that for all k ≥ k_5,

$$P\left(T_{n_{m_{k-1}}} \geq B_{n_{m_k}} (\log n_{m_k})^{\frac{\varepsilon^*-\varepsilon}{\alpha}}\right) \leq C_6 n_{m_{k-1}}\; P\left(X_1 \geq B_{n_{m_k}} (\log n_{m_k})^{\frac{\varepsilon^*-\varepsilon}{\alpha}}\right).$$

Following the steps similar to those used to get an upper bound of $P(M_k)$, one can find a k_6 such that for all k (≥ k_6),

$$P\left(T_{n_{m_{k-1}}} \geq B_{n_{m_k}} (\log n_{m_k})^{\frac{\varepsilon^*-\varepsilon}{\alpha}}\right) \leq C_7 \frac{n_{m_{k-1}}}{n_{m_k}} \frac{1}{(\log n_{m_k})^{(\varepsilon^* - \frac{3\varepsilon}{2})}}$$

By (6), we have $n_{m_k} \geq \beta^{(k-1)^\delta}$ implies $n_{m_{k+1}} \geq \beta^{k^\delta} \geq n_{m_k}$ and since $\operatorname*{Lim\,inf}_{k\to\infty} \frac{n_{k+1}}{n_k} > 1$, implies there exists λ>1 such that $n_{k+1} \geq \lambda\, n_k$. Therefore,

$$n_{m_{k+1}} \geq \beta^{k^\delta} \geq n_{m_k} \geq \lambda n_{m_{k-1}} \Rightarrow \lambda n_{m_{k-1}} \leq \beta^{k^\delta} \Rightarrow n_{m_{k-1}} \leq \frac{1}{\lambda}\beta^{k^\delta} = \lambda_1 \beta^{k^\delta}$$, where $\lambda_1 = \frac{1}{\lambda}$.

Hence $\frac{n_{m_{k-1}}}{n_{m_k}} \leq \frac{\lambda_1 \beta^{k^\delta}}{\beta^{(k-1)^\delta}} \cong \frac{\lambda_1}{\beta^{k^{\delta_1}}}$. Therefore,

$$\sum_{k=k_5}^{\infty} P\left(T_{n_{m_{k-1}}} \geq B_{n_{m_k}} (\log n_{m_k})^{\frac{\varepsilon^*-\varepsilon}{\alpha}}\right) \leq \sum_{k=k_5}^{\infty} \frac{n_{m_{k-1}}}{n_{m_k}} \frac{1}{(\log n_{m_k})^{(\varepsilon^* - \frac{3\varepsilon}{2})}} \leq \lambda_1 \sum_{k=k_5}^{\infty} \frac{1}{\beta^{k^{\delta_1}} (\log n_{m_k})^{(\varepsilon^* - \frac{3\varepsilon}{2})}} < \infty.$$

Hence $P\left(T_{n_{m_{k-1}}} \geq B_{n_{m_k}} (\log n_{m_k})^{\frac{\varepsilon^*-\varepsilon}{\alpha}} \text{ i.o}\right) = 0$, which implies the proof of (4) follows from (6) and (7) and the proof of the theorem is completed.

The proof of the theorem can be completed by noticing that $\operatorname*{Lim}_{k\to\infty} \frac{n_{k+1}}{n_k} = \infty$ comes under the class of at least geometrically increasing subsequence and proof also follows on similar lines to (2), so we omit the details.

THEOREM 2

Let $\{X_n, n \geq 1\}$ be a sequence of i.i.d. r.v.s with a common distribution function F and assume that $F \in DP(\alpha)$, $0 < \alpha < 1$. Let $\{B_n\}$ be a sequence of constants, as defined in Lemma 2, with $B_n > 0$, $T_{n_k} = \sum_{i=1}^{n_k} f\left(\frac{i}{n_k}\right) X_{n_k}$, where f is a member of BV [0,1]. Let (n_k) be an integer subsequence such that $\operatorname{Lim\,sup}_{k \to \infty} \frac{n_{k+1}}{n_k} < \infty$. Then $\operatorname{Lim\,sup}_{k \to \infty} \left(\frac{T_{n_k}}{B_{n_k}}\right)^{\frac{1}{\log\log n_k}} = e^{\frac{1}{\alpha}}$a.s .

PROOF

Proceeding as in Theorem 1, it is enough if we show that for any $\varepsilon_1 \in (0, 1)$,

$$P\left(T_{n_k} \geq B_{n_k} (\log n_k)^{\frac{1+\varepsilon_1}{\alpha}} \text{i.o}\right) = 0 \tag{8}$$

and

$$P\left(T_{n_k} \geq B_{n_k} (\log n_k)^{\frac{1-\varepsilon_1}{\alpha}} \text{i.o}\right) = 1 \tag{9}$$

One can notice that the proof of (8) follows as a consequence of theorem of Gooty Divanji and Kokkada Vidyalaxmi (2011). I.e., $\operatorname{Lim\,sup}_{k\to\infty} \left(\frac{T_{n_k}}{B_{n_k}}\right)^{\frac{1}{\log\log n_k}} \leq \operatorname{Lim\,sup}_{n\to\infty} \left(\frac{T_n}{B_n}\right)^{\frac{1}{\log\log n}} = e^{\frac{1}{\alpha}}$a.s .

From the condition $\operatorname{Lim\,sup}_{k \to \infty} \frac{n_{k+1}}{n_k} < \infty$, we see that the sequences are at the most geometrically increasing, which implies that there exists $\theta > 1$ such that

$$n_{k+1} \leq \theta\, n_k . \tag{10}$$

Now define

$$\nu_j = \min\left\{k : n_k > M^j\right\} \tag{11}$$

j=1, 2, … where M is chosen such that $\frac{\theta}{M} < 1$. Proceeding as in Allan Gut (1986) one can show that $M^j < n_{\nu_j} < \theta M^j$ and $\frac{1}{\theta M} \leq \frac{n_{\nu_{j-1}}}{n_{\nu_j}} \leq \frac{\theta}{M} < 1$. Consequently (n_{ν_j}) satisfies the condition

$\operatorname{Lim\,sup}_{j \to \infty} \frac{n_{\nu_{j-1}}}{n_{\nu_j}} < 1$ of Theorem 1 and also from the relation $\sum_{j=1}^{\infty} (\log n_{\nu_j})^{-\varepsilon_1} < \infty$, for all $\varepsilon_1 > 1$

(i.e., $\varepsilon^* = 1$). Now (9) follows from Theorem 1. Hence proof of the theorem is completed.

3. BOUNDARY CROSSING PROBLEMS

Here we study some boundary crossing random variables related to Theorem 1 and Theorem 2. Define, for any $\varepsilon > 0$,

$$Y_{n_k}(\varepsilon) = \begin{cases} 1, & \text{if } \; T_{n_k} \geq B_{n_k} (\log n_k)^{\frac{\theta-\varepsilon}{\alpha}} \\ 0, & \text{otherwise} \end{cases}$$

where $\theta = \begin{cases} \varepsilon^*, & \text{if } (n_k) \text{ is at least geometrically fast} \\ 1\;, & \text{if } (n_k) \text{ is at most geometrically fast} \end{cases}$ and

$$\varepsilon^* = \inf\left\{\varepsilon_1 > 0: \sum_{k=1}^{\infty}\left(\log n_k\right)^{-\varepsilon_1} < \infty\right\}.$$

Let for any ε > 0, $N_{m_k}(\varepsilon)$ be the partial sum sequence of $Y_{n_k}(\varepsilon)$.

I.e., $N_{m_k}(\varepsilon) = \sum_{k=1}^{m_k} Y_{n_k}(\varepsilon)$. Observe that by (8) of Theorem 2, $N_\infty(\varepsilon)$ is a proper random variable. We study this problem as Corollaries to Theorem 1 and Theorem 2. Here we show that all the moments in $0 < \lambda \le 1$ are finite for this proper random variable. This proper r.v. $N_\infty(\varepsilon)$ was studied by various authors such as Slivka, J (1969) and Slivka, J and Savero, N.C (1970).

COROLLARY 1

Let $\{X_n, n\geq 1\}$ is a sequence of i.i.d. r. v. s with a common d.f F and assume that $F \in DP(\alpha)$, $0 < \alpha < 1$. Let $\{n_k, k\geq 1\}$ be an increasing subsequence and for ε > 0 and for any λ,

$0 < \lambda \le 1$, $E\, N_\infty^\lambda < \infty$, whenever $\sum_{k=1}^{\infty} n_k^{\lambda-1} P\left(T_{n_k} > B_{n_k} (\log n_k)^{\frac{\theta+\varepsilon}{\alpha}}\right) < \infty.$

PROOF

First, we show that, for λ = 1, $E\, N_\infty(\varepsilon) < \infty$ and then claim that existence of lower moments follows from higher moments. Observe that,

$$E\, N_\infty(\varepsilon) = \sum_{k=1}^{\infty} P\left(T_{n_k} > B_{n_k} (\log n_k)^{\frac{\varepsilon^*+\varepsilon}{\alpha}}\right).$$

Following similar steps of proof of (3), we can find some constant $C_1>0$ and some $k_1>0$ such that for all $k\geq k_1$, $E\, N_\infty(\varepsilon) \le C_1 \sum_{k=k_1}^{\infty} \frac{1}{(\log n_k)^{-(\theta+\frac{\varepsilon}{2})}} < \infty$,

since $\theta = \begin{cases} \varepsilon^*, & \text{if } (n_k) \text{ is at least geometrically fast} \\ 1, & \text{if } (n_k) \text{ is at most geometrically fast} \end{cases}$ and hence proof of (4) and (10).

which claims $E\, N_\infty(\varepsilon) < \infty$, for λ=1 and therefore $E\, N_\infty^\lambda < \infty$, for λ<1. Hence the proof of the Corollary 1 is completed.

COROLLARY 2

Let $\{X_n, n\geq 1\}$ is a sequence of i.i.d. r. v. s with a common d.f F and assume that $F \in DP(\alpha)$, $0 < \alpha < 1$. Let $\{n_k, k\geq 1\}$ be an increasing subsequence of positive integers such that $\frac{n_{k+1}}{n_k} \to \infty$, as $k \to \infty$, and for ε > 0 and for any λ, $0 < \lambda \le 1$, $E\, N_\infty^\lambda < \infty$, whenever $\sum_{k=1}^{\infty} n_k^{\lambda-1} P\left(T_{n_k} > B_{n_k} k^{\frac{1+\varepsilon}{\alpha}}\right) < \infty$.

PROOF

First, we show that, for λ=1, $E\, N_\infty(\varepsilon) < \infty$ and then claim that existence of lower moments follows from higher moments. Observe that, $E\, N_\infty(\varepsilon) = \sum_{k=1}^{\infty} P\left(T_{n_k} > B_{n_k} k^{\frac{1+\varepsilon}{\alpha}}\right)$.

Following similar steps of proof of (10), we can find some constant $C_1>0$ and some $k_1>0$ such that for all $k\geq k_1$, $E\, N_\infty(\varepsilon) \le C_1 \sum_{k=k_1}^{\infty} \frac{1}{k^{1+\frac{\varepsilon}{2}}} < \infty$, which claims $E\, N_\infty(\varepsilon) < \infty$, for λ=1 and therefore

$E\, N_\infty^\lambda < \infty$, for λ<1. Hence the proof of the Corollary 2is completed.

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