



CHEMICAL REACTION AND HEAT SOURCE EFFECTS ON NONLINEAR RADIATIVE MHD BOUNDARY LAYER FLOW OF LIQUID METAL EMBEDDED IN A POROUS MEDIUM

Syam Sundar Majety¹ | Dr. K. Gangadhar² | Dr. G.V. RAMANA REDDY³

¹ Department of Nanotechnology, Acharya Nagarjuna University, Andhra Pradesh, India. (* Corresponding Author)

¹ Research Scholar, Department of Mathematics, Acharya Nagarjuna University, Andhra Pradesh, India. (* Corresponding Author)

² Department of Mathematics, Acharya Nagarjuna University, Ongole, Andhra Pradesh, India.

³ Department of Mathematics, KL University Vaddeswaram, Guntur(Dt), 502522, India.

ABSTRACT

The effects of chemical reaction on steady nonlinear MHD boundary layer flow of a viscous incompressible fluid over a nonlinear porous stretching surface embedded in a porous medium in presence of nonlinear radiation and heat generation is analyzed. The liquid metal is assumed to be gray, emitting, and absorbing but non-scattering medium. Governing nonlinear partial differential equations are transformed to nonlinear ordinary differential equations by utilizing suitable similarity transformation. The resulting nonlinear ordinary differential equations are solved numerically using Runge-Kutta fourth order method along with shooting technique. Comparison with previously published work is obtained and good agreement is found. The effects of various governing parameters on the liquid metal fluid dimensionless velocity, dimensionless temperature, dimensionless concentration, skin-friction coefficient, Nusselt number and Sherwood number are discussed with the aid of graphs.

Keywords: Chemical Reaction; Thermal Radiation; Nonlinear MHD; Porous Medium; Heat Generation; Steady Flow.

1. Introduction

The deep interest in the porous medium is easily understandable since porous medium is used in vast applications, which covers many engineering disciplines. For instance, applications of the porous media includes, thermal insulations of buildings, heat exchangers, solar energy collectors, geophysical applications, solidification of alloys, nuclear waste disposals, drying processes, chemical reactors, energy recovery of petroleum resources etc.,

Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in recent years. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, in chemical reaction engineering heat and mass transfer occur simultaneously. Chambre and Young [1] have analyzed a first order chemical reaction in the neighborhood of a horizontal plate. Muthucumaraswamy [2] derived the effects of chemical reaction, heat and mass transfer along a moving vertical surface with suction.

In recent years, a great deal of interest has been generated in the area of heat and mass transfer of the boundary layer flow over a nonlinear stretching sheet, in view of its numerous and wide-ranging applications in various fields like polymer processing industry in particular in manufacturing process of artificial film and artificial fibers and in some applications of dilute polymer solution.

The radiation effects have important applications in physics and engineering particularly in space technology and high temperature processes. But very little is known about the

effects of radiation on the boundary layer. Thermal radiation effects on the boundary layer may play important role in controlling heat transfer in polymer processing industry where the quality of the final product depends on the heat controlling factors to some extent. High temperature plasmas, cooling of nuclear reactors, liquid metal fluids, and power generation systems are some important applications of radiative heat transfer. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas. Moreover, when radiative heat transfer takes place, the fluid involved can be electrically conducting since it is ionized due to the high operating temperature. Accordingly, it is of interest to examine the effect of the magnetic field on the flow. Studying such effect has great importance in the application fields where thermal radiation and MHD are correlative. The process of fusing of metals in an electrical furnace by applying a magnetic field and the process of cooling of the first wall inside a nuclear reactor containment vessel where the hot plasma is isolated from the wall by applying a magnetic field are examples of such fields. Elbashbeshy [3] free convection flow with variable viscosity and thermal diffusivity along a vertical plate in the presence of magnetic field. Raptis and Perdakis [4] studied viscous flow over a nonlinear stretching sheet in the presence of a chemical reaction and magnetic field. Awang and Hashim [5] obtained the series solution for flow over a nonlinearly stretching sheet with chemical reaction and magnetic field. Abbas and Hayat [6] addressed the radiation effects on MHD flow due to a stretching sheet in porous space. Effect of radiation on MHD steady asymmetric flow of an electrically conducting fluid past a stretching porous sheet has been analysed analytically by Ouaf [7], Mukhopadhyay and Layek [8] investigated the effects of thermal radiation and variable fluid viscosity on free

convection flow and heat transfer past a porous stretching surface. Gururaj and Pavithra [9] investigated nonlinear MHD boundary layer flow of a liquid metal over a porous stretching surface in presence of radiation. The effects of variable viscosity and nonlinear radiation on MHD flow over a stretching surface with power-law velocity was reported by Anjali Devi and Gururaj [10].

A study on MHD heat and mass transfer free convection flow along a vertical stretching sheet in the presence of magnetic field with heat generation was carried out by Samad and Mohebujjaman [11]. Singh [12] analyzed the MHD free convection and mass transfer flow with heat source and thermal diffusion. Kesavaiah et al. [13] reported that the effects of the chemical reaction and radiation on MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in porous medium with heat source and suction. Mohammed Ibrahim and Bhaskar Reddy [14] studied the effects of thermal radiation on steady MHD free convective flow past along a stretching surface in presence of viscous dissipation and heat source. MHD heat and mass transfer free convection flow along a vertical stretching sheet in presence of magnetic field with heat generation was studied by Samad and Mobeujjaman. [15].

However the interaction of nonlinear radiation with chemical reaction of an electrically conducting and diffusing fluid past a nonlinear stretching surface has received little attention. Hence an attempt is made to investigate the nonlinear radiation effects on a steady convective flow over nonlinear stretching surface in presence of magnetic field, porous medium and heat generation. The governing equations are transformed by using similarity transformation and the resultant dimensionless equations are solved numerically using the Runge-Kutta fourth order method with shooting technique. The effects of various governing parameters on the velocity, temperature, concentration, skin-friction coefficient, Nusselt number and Sherwood number are shown in figures.

2. Mathematical Analysis

Consider coupled radiation and forced convection along a horizontal porous stretching surface which is kept at uniform temperature T_w and moving with velocity $u_w = u_0 x^m$ through and stationary liquid metal. The liquid metal is assumed to be a gray, emitting, absorbing and electrically conducting, but non-scattering medium at temperature T_∞ . A variable magnetic field is applied normal to the horizontal surface $B(x) = B_0 x^{\frac{m-1}{2}}$ in accordance with Chaim [16].

The following assumptions are made

1. Flow is two-dimensional, steady and laminar.
2. The fluid has constant physical properties.
3. The usual boundary layer assumptions are made [Ali [17]].
4. The x - axis runs along the continuous surface in the

direction of motion and y - axis perpendicular to it. and

5. The radiation dissipation in the x -axis is negligible in comparison with that in the y -axis following the lines of Michael F. Modest (Radiative Heat Transfer. Page 696) [18].

The continuity, momentum, energy conservation and mass conservation equations under the above assumption are written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \left(\frac{\sigma B^2(x)}{\rho} \right) u - \frac{\nu}{K'} u \tag{2}$$

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = K \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} + Q_0 T \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - kr'(C - C_\infty) \tag{4}$$

With the associated boundary conditions

$$u_w = u_0 x^m, \quad v = v_0(x), \quad T = T_w, \quad C = C_w \quad \text{at } y = 0$$

$$u = 0, \quad T = T_\infty, \quad C = C_\infty \quad \text{as } y \rightarrow \infty. \tag{5}$$

where u, v are velocity component of fluid in x and y direction, ρ is the density of the fluid, ν is the kinematic viscosity, σ is the electric conductivity, K is the thermal conductivity, B_0 is constant applied magnetic induction, c_p is specific heat at constant pressure, $B(x)$ is the variable applied magnetic induction, $v_0(x)$ is the variable injection velocity $\left(v_0(x) = c \sqrt{\frac{\nu u_w}{x}} \right)$, T is temperature of the fluid, C is the concentration of the fluid, T_w is the temperature of the heated surface, T_∞ is the temperature of the ambient fluid, C_w is the concentration of the heated surface, C_∞ is the concentration of the ambient fluid, q_r is the component of radiative flux, u_0 is constant, m velocity exponent parameter, kr' is the chemical reaction rate constant.

The radiative heat flux term is simplified by using the Roseland diffusion approximation (Hossian et al [19]) and accordingly

$$q_r = - \frac{16\sigma^* T^3}{3\alpha^*} \frac{\partial T}{\partial y} \tag{6}$$

where σ^* is Stefan-Boltzmann constant, α^* is the Rosseland mean absorption coefficient.

The equation of continuity is satisfied if we choose a stream function $\psi(x, y)$ such that

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}$$

Introduction the usual similarity transformation [Ali [17]].

$$\eta(x, y) = y \sqrt{\frac{m+1}{2}} \sqrt{\frac{u_0 x^{m-1}}{\nu}}$$

$$\psi(x, y) = \sqrt{\frac{2}{m+1}} \sqrt{\nu u_0 x^{m+1}} f(\eta) \tag{7}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad \theta_w = \frac{T_w}{T_\infty}$$

Equations (2) , (3) and (4) can be written as

$$f''' + ff'' - \left(\frac{2m}{m+1}\right) f'^2 - \left(M^2 + \frac{1}{K}\right) f' = 0 \tag{8}$$

$$\left[1 + \frac{4}{3R}(1 + (\theta_w - 1)\theta)^3\right] \theta'' + \frac{4}{R} \left([1 + (\theta_w - 1)\theta]^2\right) \theta'^2 + Pr f \theta' + Pr Q \theta = 0 \tag{9}$$

$$\phi'' + Sc f \phi' - Sc \gamma \phi = 0 \tag{10}$$

$$\phi'' + \frac{1}{2} Sc f \phi' - Sc Kr \phi + Sc So \theta'' = 0$$

where $M = \sqrt{\frac{2\sigma B_0^2}{\rho u_0(m+1)}}$ is the magnetic interaction parameter,

$R = \frac{K\alpha^*}{4\sigma^* T_\infty^3}$ is radiation parameter,

$K = \frac{K'u_0}{\nu}$ is permeability parameter, $Q = \frac{Q_0}{u_0}$ is heat generation parameter ,

$Sc = \frac{\nu}{D}$ is the Schmidt number,

$\gamma = \frac{\nu k r'}{u_0}$ is the chemical reaction parameter.

with the boundary conditions

$$f(0) = -S, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \phi(0) = 1 \tag{11}$$

$$f'(\infty) = 0, \quad \theta(\infty) = C, \quad \phi(\infty) = 0$$

where $S = \sqrt{\frac{2}{m+1}} c$ is the Porosity parameter, c is non dimensional constant. (for injection $S > 0$ and for suction $S < 0$).

3. Solution of the Problem

The set of coupled non-linear governing boundary layer equations (8) – (10) together with the boundary condition (11) are solved numerically by using Runge-Kutta fourth order technique along with shooting method. First of all, higher order non-linear differential equations (8) – (10) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique (Jain et.al[20]). The resultant initial value problem is solved by employing Runge-Kutta fourth order technique. The step size $\Delta \eta = 0.05$ is used to obtain the numerical solution with five decimal place accuracy as the criterion of convergence. From the process of numerical computation, the skin-friction coefficient, the Nusselt number and the Sherwood number, which are respectively proportional to $f''(0)$, $\theta'(0)$ and $\phi'(0)$, are also display in graphs.

4. Results and Discussion

As a result of the numerical calculations, the dimensionless velocity $f'(\eta)$, dimensionless temperature $\theta(\eta)$ and dimensionless concentration $\phi(\eta)$ distributions for the flow under consideration are obtained and their behavior have been discussed for variations in the governing parameters viz., the magnetic interaction parameter M , velocity exponent parameter m , permeability parameter K , Porosity parameter S , Radiation parameter R , Prandtl number Pr , surface temperature parameter θ_w , heat generation parameter Q , Schmidt number Sc and chemical reaction parameter γ . In the present study, the following default parametric values are adopted. $M = 1.0, m = 1.0, K = 1.0, S = 0.1, Pr = 0.71, R = 1.0, \theta_w = 1.1, Q = 0.05, Sc = 0.6, \gamma = 0.5$. All graphs therefore correspond to these unless specifically indicated on the appropriate graph.

In order to ascertain the accuracy of our numerical results, the present study is compared with the previous study. The temperature profiles are compared with available theoretical solution of Elbashbeshy [3], free convection flow with variable

viscosity and thermal diffusivity along a vertical plate in the presence of magnetic field in **Figure 1** and **Figure 2** It is observed that the present results are in good agreement with that of Elbashbeshy [3].

Prandtl number variation over the dimensionless temperature profile is elucidated through **Figure. 3** As Prandtl number Pr increases, the temperature $\theta(\eta)$ decreases, illustrates the face that the effect of Prandtl number Pr is to decrease the temperature in the magnetic field. Furthermore, the effect of Prandtl number Pr is to reduce the thickness of thermal boundary layer. The influence of the Schmidt number Sc on the dimensionless concentration profiles is plotted in **Figure 4** As the Schmidt number increases the concentration decreases.

The non-dimensional velocity, temperature and concentration profiles for the different values of suction S are shown through **Figures.5 (a), 5(b) and 5(c)** respectively. From these Figures it is observed that velocity, temperature and concentration profiles are increases with an increase of suction parameter S .

It is observed from **Figure 6** that the dimensionless rate of heat transfer $\theta'(0)$ increases with increase of velocity exponent parameter m . further, it is noted that the dimensionless rate of heat transfer $\theta'(0)$ increases in magnitude for increasing magnetic interaction parameter M .

Figure 7 shows the dimensionless rate of mass transfer $\phi'(0)$ increases with increase of velocity exponent parameter m . also it is observed that the dimensionless rate of mass transfer $\phi'(0)$ increases in magnitude for increasing magnetic interaction parameter M . **Figure 8** demonstrates the effect of magnetic interaction parameter M over the dimensionless rate of heat transfer $\theta'(0)$ for different values of radiation parameter R . It is seen that the dimensionless rate of heat transfer $\theta'(0)$ decreases with increase of radiation parameter R and increases with respect to magnetic interaction parameter M .

Figure 9 portrays the variation of dimensionless rate of heat transfer $\theta'(0)$ against the magnetic interaction parameter M for different values of heat generation parameter Q . it is apparent that increasing the dimensionless rate of heat transfer $\theta'(0)$ decreases with increase of heat generation parameter Q and increases with respect to magnetic interaction

parameter M . It is observed from **Figure 10** that the dimensionless rate of heat transfer $\theta'(0)$ increases with increase of permeability parameter K . further, it is noted that the dimensionless rate of heat transfer $\theta'(0)$ increases in magnitude for increasing magnetic interaction parameter M . **Figure. 11** shows the dimensionless rate of mass transfer $\phi'(0)$ increases with increase of permeability parameter K . also it is observed that the dimensionless rate of mass transfer $\phi'(0)$ increases in magnitude for increasing magnetic interaction parameter M .

Fig.1. Temperature profiles for different R

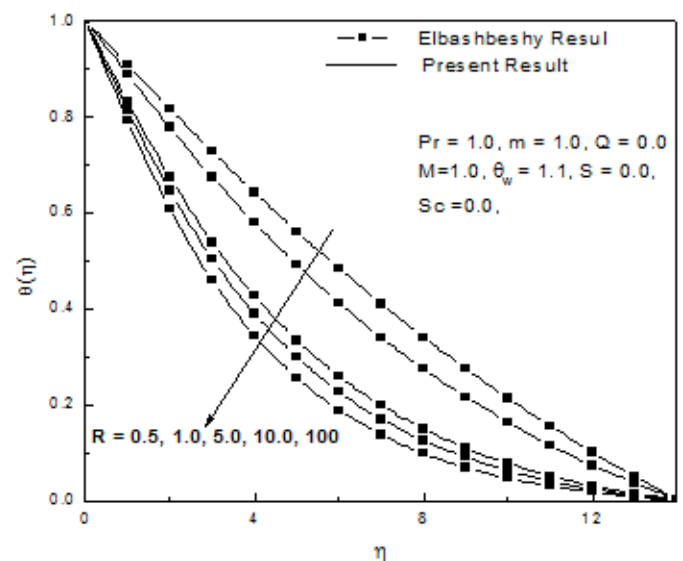


Fig.2. Temperature profiles for different θ_w

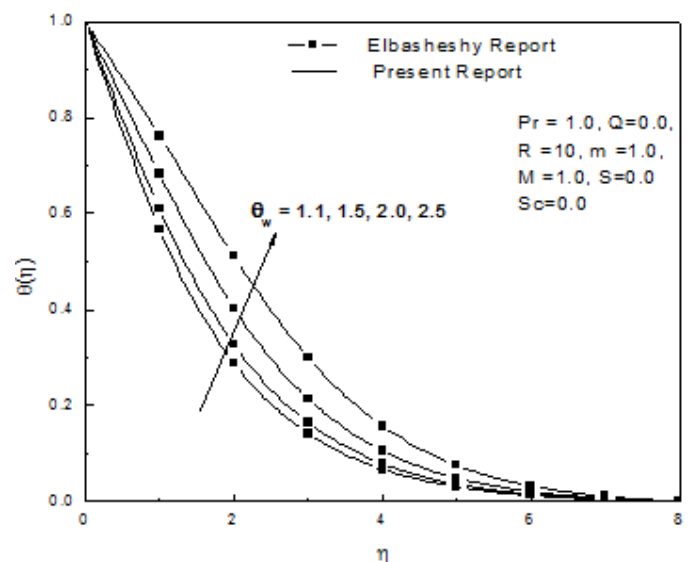


Fig. 3. Temperature profiles for different Pr

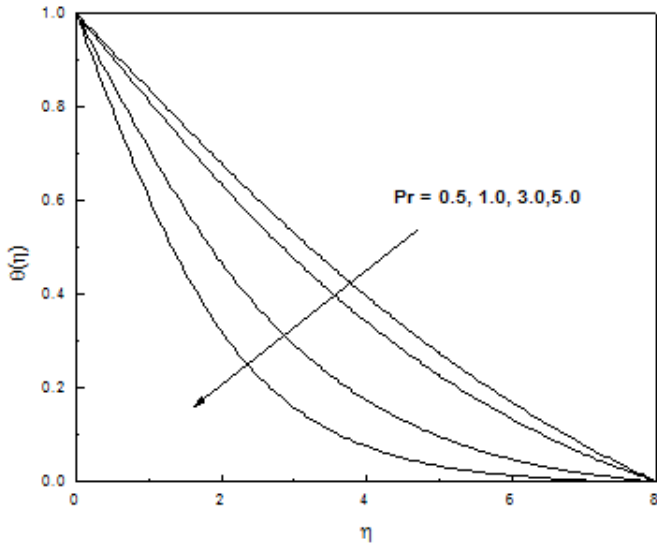


Fig. 4. Concentration profiles for different Sc

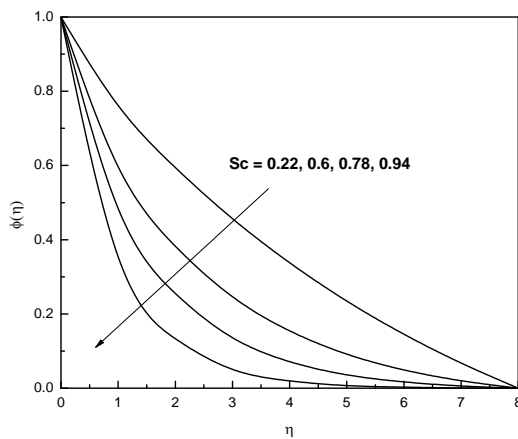


Fig. 5(a) . Velocity Profiles for different S

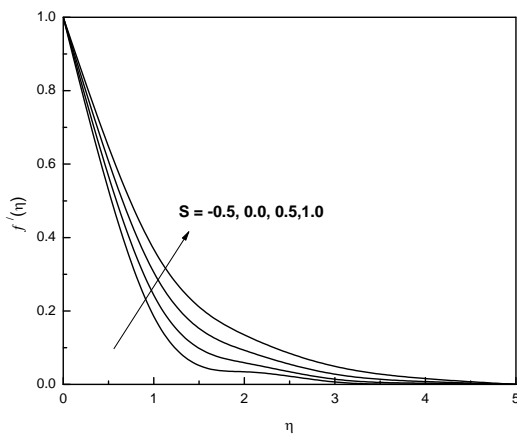


Fig. 5(b). Temperature Profiles for different S

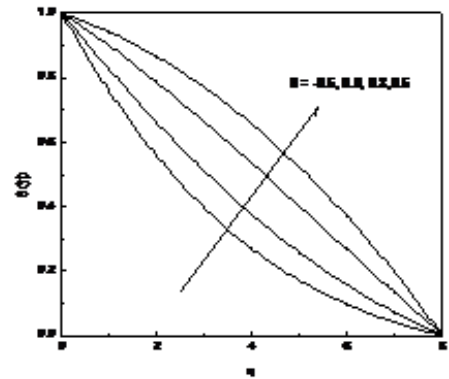


Fig. 5(c). Concentration Profiles for different S

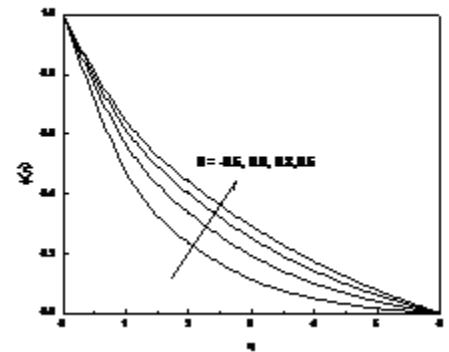


Fig.6. Dimensionless rate of heat transfer for different m

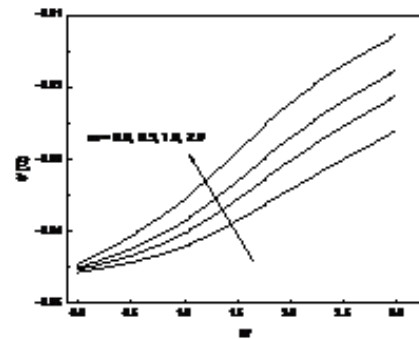


Fig.7. Dimensionless rate of mass transfer for different m

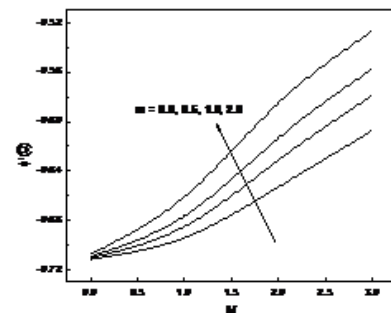


Fig.8. Dimensionless rate of heat transfer for different R

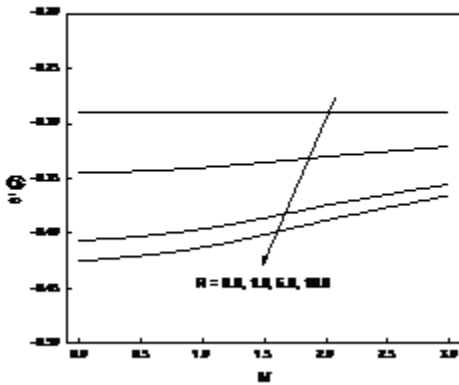


Fig.9. Dimensionless rate of heat transfer for different Q

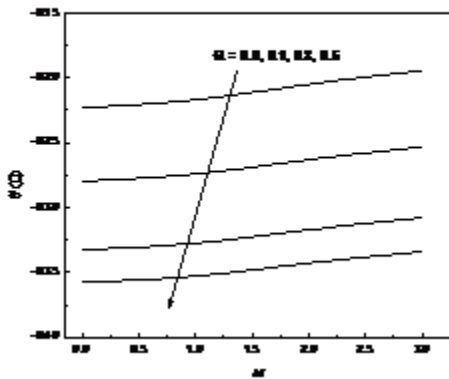


Fig.10. Dimensionless rate of heat transfer for different k

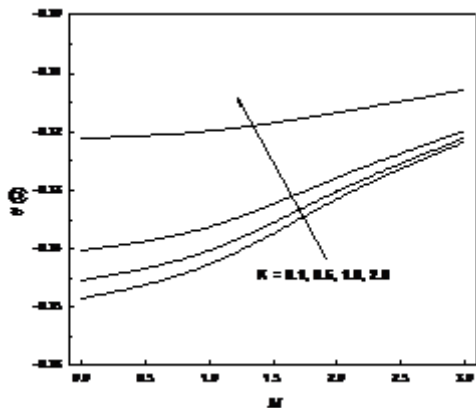
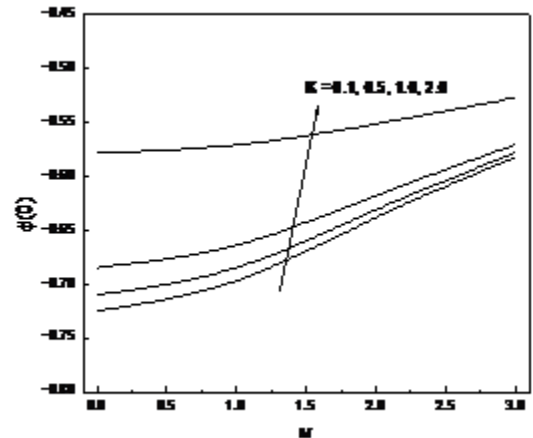


Fig.11. Dimensionless rate of mass transfer for different k



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