TRAVELING WAVE SOLUTIONS FOR WAVE-WAVE INTERACTION MODEL IN IONIC MEDIA

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ABSTRACT

The coupled one dimension nonlinear Schrödinger zakharov system (sch-zakh) is considered as the model equation for wave-wave interactions model in ionic media. The exp - expansion scheme is used to derive Traveling wave solutions for the model equations. The obtained solutions include solitons and kink solutions.

KEYWORDS: $\exp(-\phi(\xi))$ -expansion Scheme, non-linear Schrödinger zakharov system, solitons and kink solutions.

Introduction

Nonlinearity is a fascinating element of nature, today, many scientists see nonlinear science as the most important frontier for the fundamental understanding of nature. Many complex physical phenomena are frequently described and modelled by nonlinear evolution equation, so the exact or analytical solutions of the discussed nonlinear evolution equation become more and more important, which is considered not only a valuable tool in checking the accuracy of computational dynamics, but also gives us a good help to readily understand the essentials of complex physical phenomenon, e.g., collision of two solitary solutions. Looking for exact solitary wave solutions to nonlinear evolution equations has long been a major concern for both mathematicians and physicists. These solutions may well describe various phenomena in physics and other fields, such as solitons and propagation with a finite speed, and thus they may give more insight into the physical aspects of the problems. Modern theories of nonlinear science have been highly developed over the last half century.

At the classical level, a set of coupled nonlinear wave equations describing the interaction between high-frequency Langmuir waves and low-frequency ion-acoustic waves were firstly derived by Zakharov [1]. Since then, this system has been the subject of a large number of studies. In one dimension, the Zakharov equations (ZE) may be written as

$$\begin{aligned} &iE_t + E_{xx} - nE = 0, \\ &n_{tt} - n_{xx} - |E|_{xx}^2 = 0 \end{aligned} \tag{1}$$

Where E is the envelope of the high-frequency electric field, n is the plasma density measured from its equilibrium value. The system can be derived from a hydrodynamic description of the plasma [2, 3]. However, some important effects such as transit-time damping and ion nonlinearities, which are also implied by the fact that the values used for the ion damping have been anomalously large from the point of view of linear ion-acoustic wave dynamics, have been ignored in the ZE. This is equivalent to say that, the ZE is a simplified model of strong Langmuir turbulence. Thus we have to generalize the ZE by taking more elements into account. Starting from the dynamical plasma equations with the help of relaxed Zakharov simplification assumptions, and through taking use of the time-averaged two-time-scale two-fluid plasma description, the ZE are generalized to contain the self-generated magnetic field [4]. The generalized Zakharov equations (GZE) are a set of coupled equations and may be written as [5]

$$\begin{aligned} &iE_t + E_{xx} - 2\beta|E|^2 E + 2nE = 0, \\ &n_{tt} - n_{xx} + |E|_{xx}^2 = 0 \end{aligned} \tag{2}$$

where E is the envelope of the high-frequency electric field, and is the plasma density measured from its equilibrium value. This system is reduced to the classical Zakharov equations of plasma physics whenever $\beta = 0$. Due to the fact that the GZE is a realistic model in plasma, it makes sense to study the solitary wave solutions of the GZE. Recently various powerful mathematical methods such as homotopy perturbation method [6], variational iteration method [7–14], Adomian decomposition method [15] and others [16–19] have been proposed to obtain exact and approximate analytic solutions for nonlinear problems.

Description of $\exp(-\phi(\xi))$ -expansion Method:

We now present briefly the main steps of the $\exp(-\phi(\xi))$ -expansion strategy that will be applied. A PDE

$$F(u, u_t, u_x, u_y, u_{tt}, u_{xx}, u_{yy}, u_{tx} \ldots) = 0 \tag{3}$$

Where $u = u(x,t)$ is an unknown function, F is a polynomial of $u(x,t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved.

Step-1 Using a wave variable $\xi = x + y \pm ct$, c is the speed of the traveling wave the traveling wave transformation $\xi = x + y \pm ct$ eq. (3) can be converted to an ODE for $u = u(\xi)$

$$P(u, u', u'', u''' \ldots\ldots) = 0 \tag{4}$$

Where, P is a polynomial of u and its derivatives and the superscripts indicate the ordinary derivatives with respect to ξ.

Step-2 Suppose the traveling wave solution of Eq.(4) can be expressed as follows

$$u(\xi) = \sum_{i=0}^{m} A_i \{\exp(-\Phi(\xi))\}^i \tag{5}$$

Where A_i are constants to be determined such that $A_m \neq 0$ $(1 \le i \le m)\Phi = \Phi(\xi)$ and satisfied the following differential equations

$$\Phi'(\xi) = \exp\{-\Phi(\xi)\} + \mu \exp\{-\Phi(\xi)\} + \lambda \tag{6}$$

Eq. (6) gives the following solutions

Solution 1 $\mu \neq 0, (\lambda^2 - 4\mu) > 0$

$$\Phi(\xi) = In\left(\frac{-\sqrt{(\lambda^2 - 4\mu)}\tanh\left(\frac{\sqrt{(\lambda^2 - 4\mu)}}{2}(\xi + \hbar)\right) - \lambda}{2\mu}\right) \tag{7}$$

Solution 2 $\mu \neq 0, (\lambda^2 - 4\mu) < 0$

$$\Phi(\xi) = In\left(\frac{\sqrt{(4\mu - \lambda^2)}\tan\left(\frac{\sqrt{(4\mu - \lambda^2)}}{2}(\xi + \hbar)\right) - \lambda}{2\mu}\right) \tag{8}$$

Solution 3 $\mu = 0, \lambda \neq 0, and (\lambda^2 - 4\mu) > 0$

$$\Phi(\xi) = -In\left(\frac{\lambda}{\exp(\lambda(\xi + \hbar)) - 1}\right) \tag{9}$$

Solution 4 $\mu=0, \lambda\neq 0, and\, (\lambda^2-4\mu)=0$

$$\Phi(\xi)=In\left(-\frac{2(\lambda(\xi+\hbar)+2)}{\lambda^2(\xi+\hbar)}\right) \tag{10}$$

Solution 5 $\mu=0, \lambda=0, and\, (\lambda^2-4\mu)=0$

$$\Phi(\xi)=In(\xi+\hbar) \tag{11}$$

where $\hbar,\lambda,\mu$ and $A_m\neq 0$ are an arbitrary constants to be determined later. The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and the nonlinear terms appearing in Eq. (4).

Step-3 We substitute Eq. (5) into (4) and then we account the function exp (-Φ (ξ)). As a result of this substitution, we get a polynomial of exp (-Φ (ξ)). We equate all the coefficients of same power of exp (-Φ (ξ)) to zero. This procedure yields a system of algebraic equations whichever can be solved to find, $\hbar,\lambda,\mu$ and A_0, A_1, A_0..... with the aid of Maple. Substituting the values of $\hbar,\lambda,\mu$ and A_0, A_1, A_0 into Eq. (5) along with general solutions of Eq. (6) completes the determination of the solution of Eq. (2).

Application exp (−ϕ())-expansion Method for generalized Zakharov equations:-
We introduce a transformation for (GZE) eq. (2)

$$E(x,t)=U(\xi)e^{i\theta},\ \eta(x,t)=V(\xi)\ \theta=kx+\omega t,\ \xi=p(x-2kt)$$

Where k, ω and p are real constant. Put these transformation in eq. (2), we have the ordinary differential equation (ODE) for $U(\xi)$ and $V(\xi)$

$$\left.\begin{aligned} &U(\xi)(k^2+\omega)-p^2U''(\xi)+2\beta U^3(\xi)-2U(\xi)V(\xi)=0,\\ &(4k^2-1)V''(\xi)+U''(\xi)=0 \end{aligned}\right\} \tag{12}$$

Where prime denotes the differential with respect to ξ.integration of second equation of eq.(12) twice with respect to ξ.

$$V(\xi)=\frac{C-U^2(\xi)}{(4k^2-1)} \tag{13}$$

Where C is second integration constant and the first one is taken to zero. The value of $V(\xi)$ is put in first eq. (12)

$$U(\xi)\left(k^2+\omega-\frac{2C}{4k^2-1}\right)-p^2U''(\xi)+2\left(\beta+\frac{1}{4k^2-1}\right)U^3(\xi)=0 \tag{14}$$

Obtain after integrating the ODE once and setting the constant of integration equal to zero. Balancing U'' with U^3 in eq. (8) gives $m+2=3m$ i.e.$m=1$ therefore, the exp (−ϕ(ξ))-expansion Method allow us to use of the finite expansion

$$U(\xi)=a_0+a_1\exp(-\phi(\xi)),\ a_1\neq 0 \tag{15}$$

Substituting eq.(15)into eq.(14) and equating all terms with the powers in $(\exp(-\phi(\xi)))^i$, and setting each of the obtained coefficients for $(\exp(-\phi(\xi)))^i$ (i=0,1,2,3,4...)to zero, yields the set of algebraic equations for λ,μ,p,k,ω and β, we obtain

$$a_1=\pm p\sqrt{\frac{(4k^2-1)}{\beta(4k^2-1)+1}},\ a_0=a_0 \qquad \lambda=\pm\frac{2a_0}{p}\sqrt{\frac{\beta(4k^2-1)+1}{(4k^2-1)}}$$
$$\mu=\frac{2a_0[\beta(4k^2-1)+1]-[2c-(k^2+\omega)((4k^2-1))]}{2p^2(4k^2-1)} \tag{16}$$

Where p, c ,k ,β are arbitrary constants.According to Eq. (16), the travelling wave solutions of the sch-zakh Eq. (2) with the help of Eq. (13) and (15) are obtained in the following form:

Solution:-1 by equation (7), when

$$\mu\neq 0\quad \beta>0\quad \Omega=\lambda^2-4\mu=\frac{2c-(k^2+\omega)(4k^2-1)}{p^2(4k^2-1)}>0$$

We have

$$E_1(x,t)=e^{i(kx+\omega t)}\left[a_0\mp\frac{2a_0[\beta(4k^2-1)+1]-[2c-(k^2+\omega)((4k^2-1))]}{2p^2(4k^2-1)\sqrt{\Omega}\tanh\left[\frac{\sqrt{\Omega}}{2}(p(x-2kt)+\hbar)\right]\pm\frac{2a_0}{p}\sqrt{\frac{\beta(4k^2-1)+1}{(4k^2-1)}}}\right] \tag{17}$$

$$\eta_1(x,t)=\frac{c}{(4k^2-1)}-\frac{1}{(4k^2-1)}\left[a_0\mp\frac{2a_0[\beta(4k^2-1)+1]-[2c-(k^2+\omega)((4k^2-1))]}{2p^2(4k^2-1)\sqrt{\Omega}\tanh\left[\frac{\sqrt{\Omega}}{2}(p(x-2kt)+\hbar)\right]\pm\frac{2a_0}{p}\sqrt{\frac{\beta(4k^2-1)+1}{(4k^2-1)}}}\right]^2 \tag{18}$$

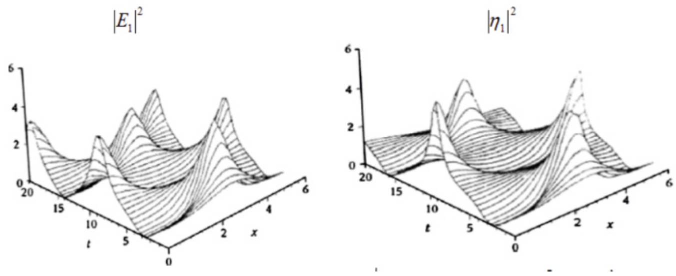

Figure: - 1 Plot of kink shaped solitary wave solution of theoretical model for $c=\beta=p=k=1, \hbar=0$

Solution:-2 by equation (8), When

$$\mu\neq 0,\ ,\ \beta>0\ \ \Omega=\lambda^2-4\mu=\frac{2c-(k^2+\omega)(4k^2-1)}{p^2(4k^2-1)}<0$$

We obtain

$$E_2(x,t)=e^{i(kx+\omega t)}\left[a_0\pm\frac{2a_0[\beta(4k^2-1)+1]-[2c-(k^2+\omega)((4k^2-1))]}{2p^2(4k^2-1)\sqrt{-\Omega}\tan\left[\frac{\sqrt{-\Omega}}{2}(p(x-2kt)+\hbar)\right]\mp\frac{2a_0}{p}\sqrt{\frac{\beta(4k^2-1)+1}{(4k^2-1)}}}\right] \tag{19}$$

$$\eta_2(x,t)=\frac{c}{(4k^2-1)}-\frac{1}{(4k^2-1)}\left[a_0\pm\frac{2a_0[\beta(4k^2-1)+1]-[2c-(k^2+\omega)((4k^2-1))]}{2p^2(4k^2-1)\sqrt{\Omega}\tan\left[\frac{\sqrt{\Omega}}{2}(p(x-2kt)+\hbar)\right]\mp\frac{2a_0}{p}\sqrt{\frac{\beta(4k^2-1)+1}{(4k^2-1)}}}\right]^2 \tag{20}$$

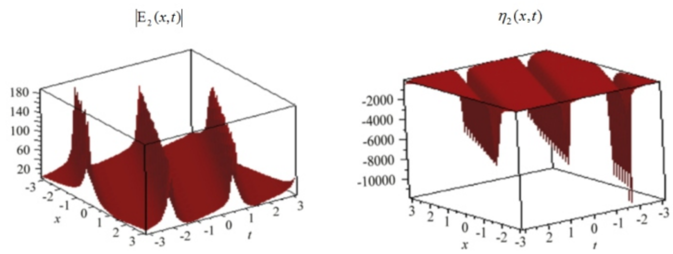

Figure 2 the wave profile of $E_2(x,t)$ and $\eta_2(x,t)$

Solution:-3 by equation (9), When

$$\mu=0,\lambda\neq 0,\ \ \Omega=\lambda^2-4\mu=\frac{2[2c-(k^2+\omega)(4k^2-1)]}{p^2(4k^2-1)}>0\ \ \beta>0$$

We obtain

$$E_3(x,t)=e^{i(kx+\omega t)}\left[\frac{2c-(k^2-\omega)(4k^2-1)}{2(\beta(4k^2-1)+1)}\pm p\sqrt{\frac{(4k^2-1)}{\beta(4k^2-1)+1}}\left\{\frac{\lambda}{\exp\lambda\{p(x-2kt)+\hbar\}-1}\right\}\right] \tag{21}$$

$$\eta_3(x,t)=\frac{c}{(4k^2-1)}-\frac{1}{(4k^2-1)}\left[\frac{2c-(k^2-\omega)(4k^2-1)}{2(\beta(4k^2-1)+1)}\pm p\sqrt{\frac{(4k^2-1)}{\beta(4k^2-1)+1}}\left\{\frac{\lambda}{\exp\lambda\{p(x-2kt)+\hbar\}-1}\right\}\right]^2 \tag{22}$$

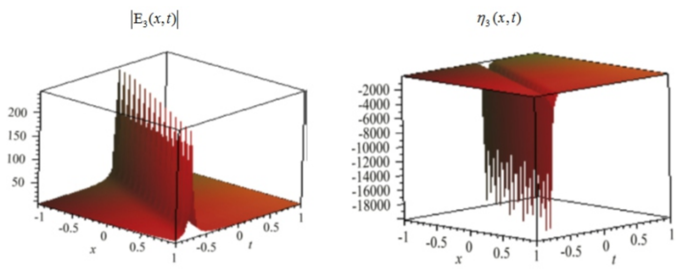

Figure 3 the wave profile of $E_3(x,t)$ and $\eta_3(x,t)$

Solution:-4 by equation (10), When $\mu\neq 0,\lambda\neq 0,(\lambda^2-4\mu)=0\Rightarrow[2c-(k^2+\omega)(4k^2-1)]=0,\beta>0$

we obtain

$$E_4(x,t)=e^{i(kx+\omega t)}\left[a_0\pm\frac{2a_0^2\sqrt{(\beta(4k^2-1)+1)}\{p(x-2kt)+\hbar\}}{2a_0\sqrt{(\beta(4k^2-1)+1)}\{p(x-2kt)+\hbar\}+p\sqrt{(4k^2-1)}}\right] \tag{23}$$

$$\eta_4(x,t)=\frac{c}{(4k^2-1)}-\frac{1}{(4k^2-1)}\left[a_0\pm\frac{2a_0^2\sqrt{(\beta(4k^2-1)+1)}\{p(x-2kt)+\hbar\}}{2a_0\sqrt{(\beta(4k^2-1)+1)}\{p(x-2kt)+\hbar\}+p\sqrt{(4k^2-1)}}\right]^2 \tag{24}$$

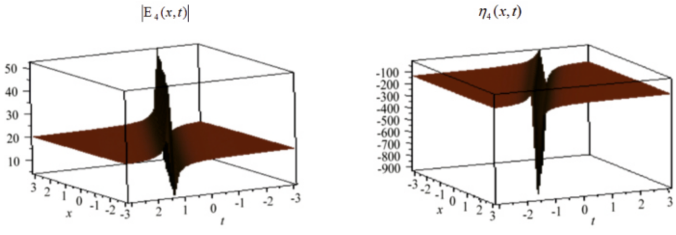

Figure 4 the wave profile of $E_4(x,t)$ and $\eta_4(x,t)$

Solution:-5 by equation (11), When $\mu\neq 0,\lambda\neq 0,(\lambda^2-4\mu)=0\Rightarrow[2c-(k^2+\omega)(4k^2-1)]=0,\beta>0$

we obtain

$$E_5(x,t)=e^{i(kx+\omega t)}\left[\sqrt{\frac{(4k^2-1)}{\beta(4k^2-1)+1}}\left(\frac{p}{[p(x-2kt)+\hbar]}\right)\right] \tag{25}$$

$$\eta_5(x,t)=\frac{c}{(4k^2-1)}-\frac{1}{(4k^2-1)}\left[\sqrt{\frac{(4k^2-1)}{\beta(4k^2-1)+1}}\left(\frac{p}{[p(x-2kt)+\hbar]}\right)\right]^2 \tag{26}$$

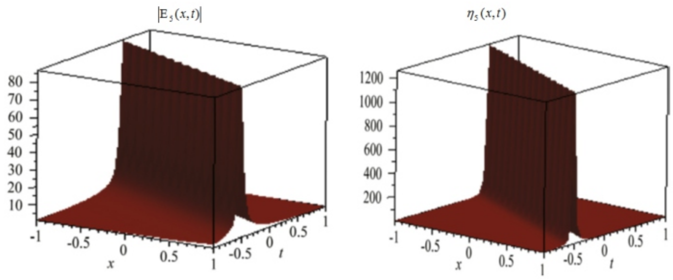

Figure 4 the wave profile of $E_5(x,t)$ and $\eta_5(x,t)$

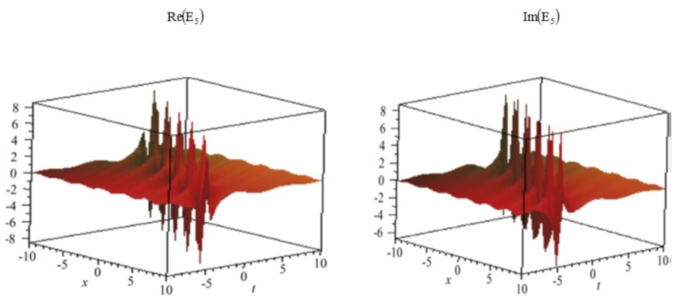

Figure 6 the wave profile of real part and imaginary part of $E_5(x,t)$

Results and discussion

The explicit exact solutions of the coupled nonlinear Eq. (2) play an important role for describing different types of wave propagation of plasma as well as fluid mechanics. The exact traveling wave solutions are obtained from the explicit solutions by choosing the particular value of the physical parameters. So, we can choose appropriate value of the physical parameters to obtain exact solutions we need in varied instances. There are various types of traveling wave solutions that are particular interest in solitary wave theory. Many author implemented different methods to this system for obtaining the solutions. To best of our knowledge, the exp (-Φ (ξ))-expansion method have not been implemented for constructing the traveling wave solutions of this model. In this paper, the traveling wave solutions $E_2,\eta_2,E_3,\eta_3,E_4,\eta_4$ and E_5,η_5 is completely new and have not found in earlier. But the solutions found in the Ref. [20] are the same to our obtain solutions $E_1(x,t)$and $\eta_1(x,t)$ respectively. By means of this scheme, we found some new travelling wave solutions of the above mentioned equations. Therefore, the exp (-Φ (ξ))-expansion method can be easily applied to solve the NLDEs and provides some new solutions. The solutions obtained in this article have been checked by putting them back into the original equation and found correct.

Conclusions

In this article, we considered complex coupled equations and the exp (-Φ (ξ))-expansion method has been successfully implemented to obtain new generalized traveling wave solutions of the one dimension nonlinear Schrödinger zakharov system (sch-zakh). The obtained solution is expressed by the hyperbolic functions, trigonometric functions and rational functions. These types of solutions have many potential applications in nonlinear optics, fluid mechanics, quantum field theory, complex scalar nucleon field, and plasma physics. It is important to point out that the exp (-Φ (ξ))-expansion method is direct, concise, elementary and comparing to other methods, like the Tanh-coth method, Jacobi elliptic function method, Exp-function method. It is easier and effective for obtaining exact solutions for a wide class of coupled nonlinear problems. We have applied this method only in coupled nonlinear equations, but it can be further applied to other coupled equations to establish new reliable solutions. The graphical representations explicitly reveal the high applicability and competence of the proposed algorithm.

REFERENCES

1. V.E. Zakharov. (1972). Collapse of Langmuir waves. Zh Eksp Teor Fiz, 62, P. 1745–1751.
2. Mv Golman. (1986). Langmuir wave solitons and spatial collapse in plasma physics. Physica, D 18, P. 67–76.
3. Michel Moisan, Jacques Pelletier. (2012): Hydrodynamic description of plasma, in: Physics of Collisional Plasmas, Springer. Netherlands.p. 203–335.
4. L.H. Li. (1993). Langmuir turbulence equations with the self-generated magnetic field. Phys. Fluids, B 5, P. 350–356.
5. B. Malomed, D. Anderson, M. Lisak, M.L. Quiroga-Teixeiro. (1977). Dynamics of solitary waves in the Zakharov model equations. Phys. Rev., E 55, P. 962–968.
6. J.H. He. (2005).Application of homotopy perturbation method to nonlinear wave equations. Chaos Solitons Fract., 26 (3), P. 695–700.
7. J.H. He, X.H. Wu. (2006). Construction of solitary solution and compaction-like solution by variational iteration method. Chaos Solitons Fract, 29 (1), P. 108–113.
8. J.H. He. (1999). Variational iteration method – a kind of nonlinear analytical technique: some examples. Int. J. Nonlinear Mech, 34 (4), P. 699–708.
9. J.H. He. (2000). Variational iteration method for autonomous ordinary differential systems. Appl. Math. Comput., 114 (2–3), P. 115–123.
10. Z.M. Odibat, S. Momani. (2006). Application of variational iteration method to nonlinear differential equations of fractional order. Int. J. Nonlinear Sci. Numer. Simul., 7 (1) P. 27–36.
11. Mehmet Giyas Sakar, Fevzi Erdogan, Ahmet Yıldırım. (2012). Variational iteration method for the time-fractional Fornberg– Whitham equation. Comput. Math. Appl., 63 (9), P. 1382– 1388.
12. J.H. He, Guo-Cheng Wu, F. Austin. (2010). The variational iteration method which should be followed. Nonlinear Sci. Lett. A–Math. Phys. Mech., 1 (1), P. 1–30.
13. Arun Kumar. (2005). A variational method to study the Zakharov equation. Int. J. Math. Math. Sci., 22, P. 3703–3709.
14. D. Kaya, S.M. El-Sayed. (2004). A numerical simulation and explicit solutions of the generalized Burger–Fisher equation. Appl. Math. Comput., 152, P. 403–413.
15. Pankaj, R. D. (2013)Laplace - Modified Decomposition Method to Study Solitary Wave Solutions of Coupled Nonlinear Klein-Gordon Schrödinger Equation, International Journal of Statistika and Mathematika, 5(1), pp 01-05
16. J.H. He. (2006). some asymptotic methods for strongly nonlinear equations. Int. J. Modern Phys., B 20, 10, P. 1141–1199.
17. E.M. Abulwafa, M.A. Abdou, A.A. Mahmoud. (2006). The solution of nonlinear coagulation problem with mass loss, Chaos Solitons Fract., 29 (2), P. 313–330.
18. A. Bekir. (2012). Multisoliton solutions to Cahn–Allen equation using double exp-function method. Phys. Wave Phenom., 20 (2), P. 118–121.
19. Abdul-Majid Wazwaz. (2008). the tanh–coth and the sine–cosine methods for kinks, solitons, and periodic solutions for the Pochhammer–Chree equations. Appl. Math. Comput., 195 (1), P. 24–33.
20. Kumar Arun, Pankaj R. D. (2015). Tanh–coth scheme for traveling wave solutions For Nonlinear Wave Interaction model. Journal of the Egyptian Mathematical Society, 23 (2), P. 282–285.