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GROUP THEORY AND GENERAL SOCIAL BEHAVIOUR: A GROUP-ACTION FRAMEWORK FOR FAMILY UNITY, SOCIAL COORDINATION AND COMMUNITY COHESION

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ABSTRACT:

This paper proposes a mathematically disciplined framework for interpreting selected principles of Group Theory in the study of general social behaviour, with particular emphasis on family coordination, unity and community cohesion. The central proposal is a group-action model in which a social state space S contains configurations of trust, communication, responsibility, cooperation and shared norms, while a set G of admissible social transformations acts on S. Closure is interpreted as the ability of compatible cooperative actions to produce another admissible cooperative state; associativity represents consistency in the composition of compatible social actions; identity represents a neutral or baseline coordinating rule that preserves the existing social state; and inverse represents a reversible restoration of a prior compatible social configuration. The paper emphasizes that the raw set of family members is not automatically a mathematical group. Rather, the group-theoretic object is constructed from admissible transformations or relations acting on the social state space. This distinction avoids an incorrect direct identification of people with algebraic elements. Family examples are used to illustrate shared decision-making, care responsibilities, financial coordination, conflict resolution and intergenerational support. The framework is connected with established social-capital and social-cohesion literature, which identifies trust, reciprocity, shared identity, norms and cooperative interaction as important dimensions of cohesive groups. The proposed model is conceptual and requires empirical validation through family- or community-level data before causal claims can be made.

KEYWORDS:

GROUP THEORY, GROUP ACTION, GENERAL SOCIAL BEHAVIOUR, FAMILY COORDINATION, SOCIAL COHESION, SOCIAL CAPITAL, TRUST, RECIPROCITY, COLLECTIVE EFFICACY, MATHEMATICAL SOCIOLOGY, SOCIAL NETWORKS.

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1. INTRODUCTION

Social groups are organized through repeated interaction, shared expectations, cooperation, conflict resolution and mechanisms for maintaining collective functioning. Families provide an especially clear setting in which these processes can be observed because members share responsibilities while also retaining individual preferences and identities. Research on social capital has long connected trust, reciprocity, networks and norms with collective action and family functioning. Recent scholarship defines social cohesion through belonging, solidarity and shared values, while also emphasizing trust and cooperation. [1,2]

Mathematical sociology provides a broader precedent for representing social processes through formal structures. A recent review emphasizes that mathematical sociology uses different mathematical programs for action, relations, organizations, status, influence, trust and collective behaviour; it also stresses that mathematical notation does not by itself supply the substantive social meaning of a model. [3] This observation is important for the present research.

The objective of this paper is therefore not to claim that society is literally a group in the abstract-algebraic sense. Instead, the paper develops a group-action interpretation of social coordination. The model asks: Can recurring cooperative transformations in a family or community be represented as algebraic transformations, and can the four group properties provide a disciplined language for studying social stability and coordination?

2. CONCEPTUAL CORRECTION: UNIVERSE, SET AND GROUP

In elementary Group Theory, the universe or underlying set is not one of the four group axioms. A group is a set G together with a binary operation satisfying closure, associativity, existence of an identity element and existence of inverses. In a social application, it is therefore preferable to distinguish three objects:

U = *the universe of relevant social states or actors*

$S \subseteq U$ = *the selected social-state space*

G = *a set of admissible transformations acting on* S

This distinction is central to the proposed model. A family F may contain parents, children and other members, but F itself is not automatically a group. Instead, transformations such as cooperative decision-making, responsibility exchange, conflict repair and restoration of agreed routines can be represented as elements of a transformation set.

3. RELATIONSHIP TO EXISTING SOCIAL-COHESION RESEARCH

Social-capital research identifies trust, norms of cooperation, shared values and relational resources as mechanisms that support cohesion. A 2026 review describes social cohesion through belonging, solidarity and shared values and links social capital with trust and norms of cooperation that can produce collective efficacy. [1] Family research similarly treats the family as a source, user and builder of social capital, highlighting trust, cooperation, coordination, reciprocity and interdependence. [4]

Research on interpersonal systems also emphasizes coordination: coordinated behaviour emerges through coupling among interacting components and can contribute either to conflict or social cohesion depending on the interaction pattern. [5] These findings motivate the present mathematical abstraction, but they do not establish that Group Theory itself causes stronger families. The proposed model is a formalization to be tested.

4. PROPOSED GROUP-THEORETIC SOCIAL BEHAVIOUR MODEL

Let S denote the social-state space. A state may be represented as a vector:

$$s = (T, C, R, N, I, P)$$

where T denotes trust, C communication quality, R reciprocal responsibility, N adherence to shared norms, I shared identity and P participation. These variables may be normalized to a chosen interval such as [0,1] for empirical modelling.

Let G be the set of admissible social transformations. Each g in G maps one social state to another:

$$g : S \to S$$

$$g \cdot s = s'$$

A transformation may represent a coordinated family meeting, a redistribution of a household responsibility, a conflict-repair protocol, a shared decision process, or a community cooperation intervention.

5. CLOSURE PROPERTY IN SOCIAL BEHAVIOUR

$$g_1, g_2 \in G \quad \Rightarrow \quad g_1 \circ g_2 \in G$$

Closure means that combining two admissible social-coordination transformations results in another admissible transformation. For example, suppose a family has a weekly planning meeting and a shared responsibility schedule. If both practices are defined as admissible coordination transformations, their compatible composition can produce a broader household-coordination state.

Example: Let g_1 represent a shared financial-planning meeting and g_2 represent allocation of household responsibilities. Then $g_1 \circ g_2$ represents a coordinated family-management routine, provided that the combined procedure remains acceptable to the members and does not violate safety, autonomy or other constraints.

The social interpretation of closure is therefore not 'everyone must agree with everyone'. It is the existence of a defined set of compatible cooperative actions whose composition remains within the agreed behavioural framework.

6. ASSOCIATIVITY IN FAMILY COORDINATION

$$(g_1 \circ g_2) \circ g_3 = g_1 \circ (g_2 \circ g_3)$$

Associativity expresses consistency in how compatible actions are composed. Consider three family processes: g_1 = information sharing, g_2 = joint decision-making, and g_3 = implementation of the decision. If these transformations are formally defined so that both compositions produce the same final social configuration, then the coordination process is associative.

In practical family life, this does not mean that human conversations are automatically associative. Rather, associativity is a design condition for a formal coordination protocol. It can help identify procedures where the order of grouping compatible sub-processes does not create contradictory outcomes.

7. IDENTITY ELEMENT: WHO OR WHAT SHOULD BE THE UNITY ELEMENT?

This is a critical point. The identity element should generally NOT be assigned to one particular family member. Treating the father, mother, eldest child or any other individual as the mathematical identity element would confuse a person with an algebraic operation.

$$e \cdot s = s$$

The identity element should instead represent a neutral baseline coordination rule or shared family constitution that preserves the current admissible state. Examples include a mutually accepted principle such as 'every member is heard before a major family decision' or 'agreed responsibilities remain unchanged unless the family jointly revises them'.

Thus, the identity element is a property of the coordination system, not a status ranking among people. In a pluralistic family, this interpretation also protects individual agency: unity does not require one person to control the others.

8. INVERSE PROPERTY AND CONFLICT REPAIR

$$g^{-1} \circ g = g \circ g^{-1} = e$$

An inverse represents a compatible restoration transformation. Suppose a family temporarily changes its responsibility allocation because one member is ill, travelling or facing an unusual workload. Let g denote the temporary reallocation. If the original allocation can be restored through a defined transformation g^{-1}, then the

pair can be represented algebraically.

Similarly, a conflict-repair process may be modelled as a transformation that moves a social state away from a high-conflict configuration toward a mutually agreed baseline. However, human relationships are often path-dependent and information may be lost. Therefore, not every social action has a true inverse. A strict group should only contain transformations for which an inverse is genuinely defined.

9. FAMILY AS A SOCIAL TRANSFORMATION SYSTEM

Consider a family with members $F=\{f_1,f_2,...,f_n\}$. Instead of assigning each member an algebraic role, define the family state as a vector:

$$s_F = (T,C,R,N,I,P)$$

Suppose the family adopts the following admissible transformations:

Transformation	Meaning	Expected state effect
g_1	Family communication meeting	May improve C and T
g_2	Responsibility redistribution	May improve R and reduce overload
g_3	Conflict-resolution protocol	May improve T, C and N
g_4	Shared family activity	May strengthen I and participation P
g_5	Emergency temporary arrangement	Changes selected responsibilities for a limited period

10. FAMILY COORDINATION EXAMPLE

Consider a household in which two adults have employment responsibilities and children have educational responsibilities. A weakly coordinated state may be represented abstractly by low communication and low reciprocity. The family can define a weekly coordination transformation g_1 that includes a short meeting, transparent sharing of important information and allocation of responsibilities.

If a second transformation g_2 creates a fair task-sharing schedule, the combined operation $g_1 \circ g_2$ creates a more structured coordination process. The model does not assume that the result will always improve family functioning; the actual effect must be measured. Candidate indicators include perceived trust, communication frequency, reciprocal support, conflict frequency and participation in joint decisions.

11. UNITY WITHOUT UNIFORMITY

A central implication of the model is that unity does not mean identical opinions. A group can maintain a common coordination structure while allowing different individual preferences. Mathematically, the shared identity element preserves the baseline coordination state, while individual transformations can represent legitimate differences.

For example, family members may have different educational, occupational or recreational preferences. The family remains coordinated when members follow agreed procedures for decisions, responsibilities and mutual support. The model therefore distinguishes:

- Unity = compatibility of the coordination framework.
- Uniformity = identical preferences, which is not required.
- Identity = the neutral shared baseline that preserves the accepted state.
- Coordination = composition of compatible actions.
- Conflict repair = transformation toward an agreed admissible state.

12. SOCIAL GROUP AND COMMUNITY EXTENSION

Let S_C denote the state space of a community. Its state vector may contain trust, participation, cooperation, shared norms, perceived fairness and inter-group communication. Community interventions can then be represented as transformations:

$$g : S_C \to S_C$$

Examples include neighbourhood meetings, community problem-solving, voluntary service coordination, transparent resource allocation and cooperative emergency planning. Research on social cohesion emphasizes trust, shared identity, basic norms and cooperative interaction as dimensions of cohesion, making these variables suitable candidates for empirical operationalization. [1,6]

The framework can also distinguish bonding and bridging processes. Strong internal family relationships may strengthen bonding, while constructive links between families, schools, workplaces and community institutions may create bridging connections. Existing social-capital research explicitly discusses structural and relational dimensions of these connections. [2,4]

13. A MATHEMATICAL SOCIAL-COHESION INDEX

For empirical testing, define a normalized social-cohesion score:

$$SC(s) = w_1 T + w_2 C + w_3 R + w_4 N + w_5 I + w_6 P$$

where $w_i \geq 0$ and the weights satisfy $\Sigma w_i = 1$. The weights should be established from theory, expert elicitation or statistical validation rather than chosen merely to produce a desired result.

For a transformation g, define its measured change as:

$$\Delta SC(g,s) = SC(g \cdot s) - SC(s)$$

A positive value indicates improvement in the selected index for that observation and measurement design. It does not prove that the transformation causes long-term social cohesion.

14. GROUP ACTION AND SOCIAL STABILITY

The proposed framework can represent social stability as the preservation of an admissible state under identity and compatible transformations. A stable family is not defined as one that never changes. Instead, stability means that changes can be coordinated and that the system has mechanisms for returning to shared norms after temporary disturbances.

$$e \cdot s = s$$

$$g^{-1}(g \cdot s) = s \quad (\text{when } g^{-1} \text{ exists})$$

This gives a mathematical language for resilience: disturbance → coordinated transformation → recovery/restoration.

15. PROPOSED SOCIAL-COORDINATION ALGORITHM

Step 1. Measure or assess the current social state s.

Step 2. Identify the relevant coordination problem.

Step 3. Define admissible transformations G consistent with autonomy, safety and agreed norms.

Step 4. Test whether the proposed transformation set satisfies the required algebraic properties.

Step 5. Apply or simulate a candidate transformation g.

Step 6. Measure the resulting state g·s.

Step 7. Calculate the change in the selected social-cohesion indicators.

Step 8. Retain, modify or reverse the intervention according to evidence and the group's agreed decision process.

16. RESEARCH HYPOTHESES

H1: A transformation-based mathematical representation can formally describe selected coordination processes within families and social groups.

H2: Families with clearly defined shared coordination rules can be represented by a more stable set of admissible social transformations than families without such rules.

H3: Trust, communication, reciprocity, shared norms, identity and participation can be operationalized as measurable state variables for a group-action social model.

H4: Restoration mechanisms corresponding to compatible inverse transformations may contribute to resilience after temporary disruptions.

These hypotheses require empirical testing and are not presented as established causal facts.

17. PROPOSED EMPIRICAL STUDY

A future study could survey families or community groups and measure the six state variables T,C,R,N,I,P. Participants could report communication quality, trust, reciprocal support, shared decision-making, common norms and participation. The same groups could then be observed across a defined period to test whether specified coordination interventions correspond to measurable changes in the state vector.

- Sample: clearly defined family or community population.
- Instrument: validated or pilot-tested questionnaire with transparent scoring.
- Observation: baseline and follow-up measurement.
- Model: estimate s_before and s_after.
- Analysis: calculate ΔSC and test associations with coordination interventions.
- Robustness: examine alternative weights and subgroup differences.
- Ethics: informed consent, confidentiality and protection of participants.

18. DISCUSSION

The main theoretical value of the framework is the separation between social entities and mathematical transformations. People are not reduced to algebraic symbols; rather, recurring coordination procedures are formalized. This preserves the social meaning of trust, autonomy and relationships while allowing mathematical analysis.

The model also aligns with the broader mathematical-sociology principle that formalization must specify the social units, state variables, transition rules and observations rather than merely adding equations to qualitative descriptions. [3]

The model should therefore be treated as a conceptual bridge between abstract algebra and mathematical sociology. It becomes a scientific model only after its variables, transformations and measurement procedures are operationally defined and tested with data.

19. LIMITATIONS

- Human social behaviour is context-dependent and cannot be fully represented by a small algebraic structure.
- Not every social interaction is reversible; therefore a strict group may apply only to a restricted set of transformations.
- Closure may fail when two individually acceptable interventions conflict when combined.
- Associativity is a property of the defined mathematical operation, not an assumption about human psychology.
- An identity element represents a neutral operation or shared baseline, not a dominant family member.
- The framework does not imply that stronger cohesion is always desirable in every circumstance; autonomy, dissent and healthy boundaries can also be important.
- Empirical validation is required before claiming effectiveness.

19. NOVELTY, POSITIONING AND ORIGINALITY

A literature search shows that mathematical sociology is an established interdisciplinary field and that prior scholarship has explicitly connected Group Theory with social-science theory. In particular, the 2017 paper "Linking Group Theory to Social Science Game Theory" develops a systematic connection between social groups, organizations, social-rule systems and interaction/game theory. Mathematical-sociology literature also includes formal models of social institutions, networks, cooperation and social behaviour. [8–10]

Accordingly, the present manuscript does not claim that Group Theory has never previously been applied to social science. Its proposed contribution is narrower and more testable: a group-action representation in which (i) a family or community is represented through a social-state space S, (ii) admissible coordination procedures form a transformation structure G, (iii) the identity element is interpreted as a neutral shared coordination rule rather than a person, and (iv) reversible interventions are explicitly distinguished from irreversible social processes. This formulation is designed to connect abstract algebraic properties with measurable family-coordination variables such as trust, communication, reciprocity, shared norms, identity and participation.

The manuscript therefore positions itself as a conceptual mathematical model that can be empirically tested, rather than as a claim of a previously unknown use of Group Theory in sociology. This positioning reduces the risk of an overstated novelty claim and makes the research question suitable for a mathematical-sociology or interdisciplinary modelling journal.

19.1 ORIGINALITY SAFEGUARDS

The paper has been written in original wording, with standard mathematical definitions and previously published concepts explicitly attributed. Standard terms such as closure, associativity, identity, inverse, group action, social capital and social cohesion are not proprietary ideas and may necessarily overlap with established literature. The manuscript avoids presenting those standard concepts as new discoveries.

19.2 PROPOSED CONTRIBUTION STATEMENT

Proposed contribution: "We develop and formalize a group-action framework for representing family and community coordination as transformations on a social-state space, with explicit conditions under which closure, associativity, identity and inverse are meaningful. We further propose measurable state variables and a validation protocol for testing the framework empirically."

19.3 SIMILARITY AND PLAGIARISM LIMITATION

No web-indexed source located in the preliminary search was found to reproduce the exact title and the complete proposed framework of this manuscript. This is not equivalent to a Turnitin or iThenticate percentage, because subscription databases, institutional repositories and unpublished manuscripts may not be fully web-indexed. A formal similarity report should therefore be generated using the journal, university or publisher's approved similarity-checking service before submission.

20. CONCLUSION

This paper develops a Group-Theoretic framework for interpreting general social behaviour through transformations acting on social states. The family is treated as a coordinated social system rather than as a set that is automatically a mathematical group. The underlying social universe supplies the possible states and actors; the transformation set supplies the algebraic structure.

Closure describes compatible composition, associativity describes consistent composition, identity represents a neutral shared baseline, and inverse represents restoration when genuine reversibility exists. The identity element should therefore be understood as a shared coordinating principle or baseline operation—not as a particular family member.

The proposed model provides a structured mathematical language for analysing family coordination, trust, reciprocity, communication, shared identity and community cooperation. Its scientific value will depend on empirical testing. The next stage should be a pilot study with measurable variables and a predefined transformation protocol, followed by statistical analysis and comparison across different family or community settings.

AUTHOR STATEMENT

The manuscript presents a conceptual mathematical framework. It does not claim that Group Theory has been empirically proven to strengthen families or societies. Such claims should be made only after an appropriately designed empirical study.

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